1 Ready-Made Materials
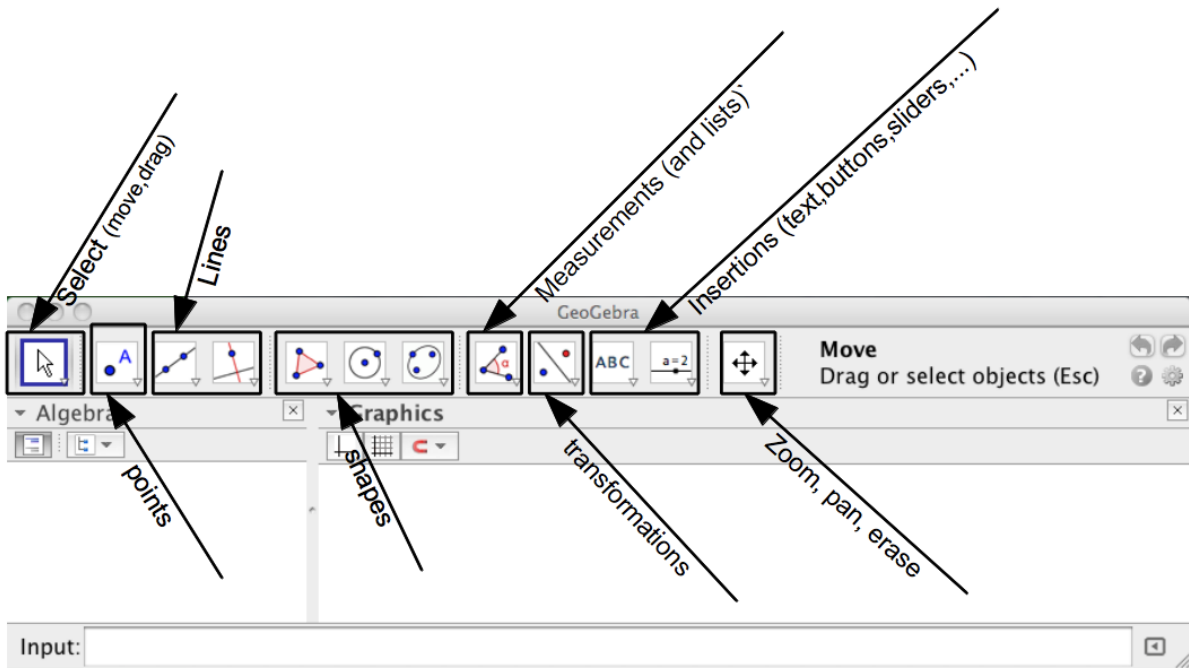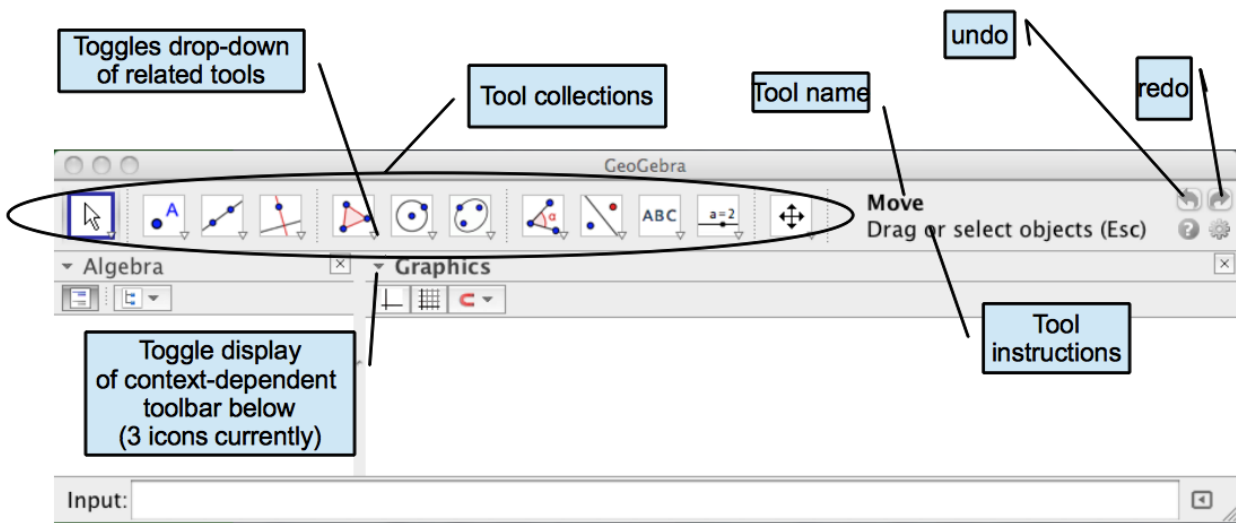
- GeoGebra Tube:

  - main site: www.geogebratube.org, search for:
    * physics
    * optics
    * projectile
    * kinematics
    * electric

  - a few things I've posted: http://www.geogebratube.org/user/profile/id/4708

- Daniel Mentrard: http://dmentrard.free.fr/GEOGEBRA/Sciences/Excelelabo.htm

- http://math247.pbworks.com/w/page/20517613/Physics%20with%20GeoGebra

2 Build Your Own

2.1 Basic Navigation

Toggles drop-down of related tools

Tool collections

Tool name

undo

redo

GeoGebra

Move
Drag or select objects (Esc)

Algebra

Graphics

Toggle display of context-dependent toolbar below (3 icons currently)

Tool instructions

Input:

Select (move,drag)

Lines

Measurements (and lists)`

Insertions (text,buttons,sliders,...)

GeoGebra

Move
Drag or select objects (Esc)

Algebra

Graphics

points

shapes

transformations

Zoom, pan, erase

Input:

Tips:

- The `esc` key changes your tool to the arrow (for moving or selecting).

- Zooming and panning can also be done with mouse and control keys

    - mouse scroll up and down zooms in and out
    - shift drag anywhere but an axis pans
    - shift drag on an axis rescales that axis

- Control-click or right-click or two-finger-tap on objects to access a contextual menu including Object Properties. I'm going to call this ctrl-click even though that may not be what you actually do (and isn't what I actually do on my laptop the way I have my trakcpad configured).

- In the Algebra View, click on the circle left of each object to toggle between visible (circle is blue) and hidden (circle is empty).

- Capitalization matters. Agecoefficient≠agecoefficient.

- If you create an object and can't find it in either the Graphics or the Algebra view, ctrl-click any object and select Object Properties. Then find your missing object in that control panel. It may be off-screen, buried, or hidden.

- The different views can be resized by dragging on boundaries marked with a dot in the middle.

- The different views can be rearranged by clicking on the view title bar and dragging it. An outline shows where the view will land if you let go at any particular mouse location.

2.2 Geometric Optics

2.2.1 Useful Tools

- point - A point placed in empty space will be moveable. These points are (usually) bright blue. A point placed on an axis, line, function, or conic section will move only along that axis, line,.... These points are (usually) light blue. Points which cannot be moved (usually) default to dark gray.

- intersection - Click the two objects that intersect. If they intersect twice, you will get two points. Hide the one you don't want. When there are infinitely many intersections (sin and cos for instance), a nearby intersection is picked; there may be a delay while GeoGebra figures out which intersection is closest. If the one you want doesn't get picked, undo and try a different location.

- midpoint - Select this tool and then select two points or a line segment and GeoGebra will create a point in the middle.

- line - Select the line tool and then select any two points to define that line. If either or both of the points move, the line will follow. The line will always run off the edges of the screen to indicate that it is infinite.

- line segment - works the same way a line does except that the ends are defined by the points.

- ray - A ray starts at the first point chosen and pass through and beyond the second point chosen. Does not have an arrowhead on it. If we want arrowheads, we need to use the vector tool .

- circular arc - Click the center of the circle the arc is a piece of first. Then click two more points to define the ends of the arc. The second point defines the radius of the circle.

2.2.2 Simple Ray Diagram

- If the axes aren't showing, show them.

  - method 1: Click the icon under the window label "Graphics" that looks like axes. If the icon isn't visible, click the triangle next to the window label "Graphics".
  - method 2: Ctrl-click on the graphics window somewhere in the middle (not a border or the title bar) and select the axes icon in the resulting pop-up menu.
  - method 3: Click the gear-like icon in the group of 4 icons on the right side of the window and then choose graphics.

- Draw a mirror.

  - For aesthetic purposes, set the mirror so it crosses the x-axis at the origin. To do so, create a point at the origin. To make a point that will definitely stick at the origin, type `Intersect[xAxis,yAxis]` in the Input bar and hit the return key. You can also do this by selecting the point tool and then clicking the intersection of the axes. If you type `(0,0)` in the Input bar, the point is a moveable point because is it just two coordinates. Notice that GeoGebra names the points in alphabetical order with Capitol letters. If you don't like the default name, ctrl-click and choose rename. You can also name the point when you create it HOWEVER, if the name you choose when you create the object starts with a lowercase letter, you will get a vector instead of a point.
  - Create another point on the x-axis to be the center of curvature of your mirror.
  - Select the arc tool. Select your center of curvature point. Select the point at the origin. Create a third point above the axis. Notices that the arc draws as you move the mouse so you have a sense of how much arc you are creating. For me, this third point is called C.
  - We'd like the mirror to be symmetric above and below the optical axis which will coincides with the x-axis. To accomplish this, do ONE of the following things.

    * In the input bar type `(x(C),-y(C))` and then hit return.

    * Select the reflection tool . Click point C and then click the x-axis.
    * In the input bar type `ref` at which point command completion will offer you three options. Use arrow keys or the mouse to select `\reflect[<Object>,<Line>]` and hit return to put that in the Input bar for you to fill in the `<>` pieces. Then `<Object>` should be highlighted. Type `C`. Then hit the tab key. Now `<Line>` should be highlighted. Type `xAxis`. Hit return.

– Use the arc tool again to draw a second arc. Note that if you pick your points in the wrong order, you will get the complement of the arc you wanted. Undo and try again if you make this mistake. This looks fine but being in two pieces may cause trouble latter (points attached to one half can't jump to the the other half). Delete both arcs but don't delete the points that define the ends of the arc. Now use the arc tool to create a single arc from C to C'. (Don't click on A as you pass by.)

- Create the focal point.

  – Select the midpoint tool and then click on the center of curvature of the mirror and the origin (in any order). This is the focal point.

  – Since this point depends entirely on other things for its location, it is gray. You can move it only by changing the center of curvature.

- Create the object. We'll use the traditional upright arrow for now.

  – Create a point somewhere above the x-axis. This will be a free point so we can move the object and change the object's height. This will be the top of the object.

  – We want the base of the object to be directly below the tip and on the optical axis. Assuming the point at the top of your object is called E, type `(x(E),0)` in the Input bar and hit return. This creates a point with the same x-coordinate as point E and 0 for the y-coordinate.

  – Use the Vector tool to draw an arrow starting at F on the axis and going up to E.

- Create the first ray.

  – We'll do the parallel ray. We need a line through E and parallel to the horizontal axis. Do ONE of the following.

    * From the right-hand collection of line tools, select the Perpendicular Line tool . Then click on point E and after that click on the y-axis.

    * From the right-hand collection of line tools, select the Parallel Line tool . The click on point E and after that click on the x-axis.

    * In the Input bar, type `lin` command completion will supply four options. Arrow down to `Line[ <Point>, <Parallel Line> ]` and hit return. Then type `E`, hit the tab key, and type `xAxis` (note capitalization), then hit return.

  – Select the Intersect Two Objects tool and then click on the mirror and the horizontal line you just drew (in either order). This point is gray because we can't move it independently. You can accomplish the same thing in the Input bar after noticing that the horizontal line we added has been named a by GeoGebra and that the mirror was called c by typing `Intersect[a,c]` and then hit return.

  – Light coming in parallel to the optical axis reflects through (or nearly so) the focal point so make a line connecting the point we just created on the mirror with the focal point. Use the Line tool rather than the Segment tool.

- Use similar techniques to draw at least one more ray.

- Create the tip of the image as the intersection of two rays.

- Create the image arrow the same way we created the object arrow.

- The final result should look something like this.
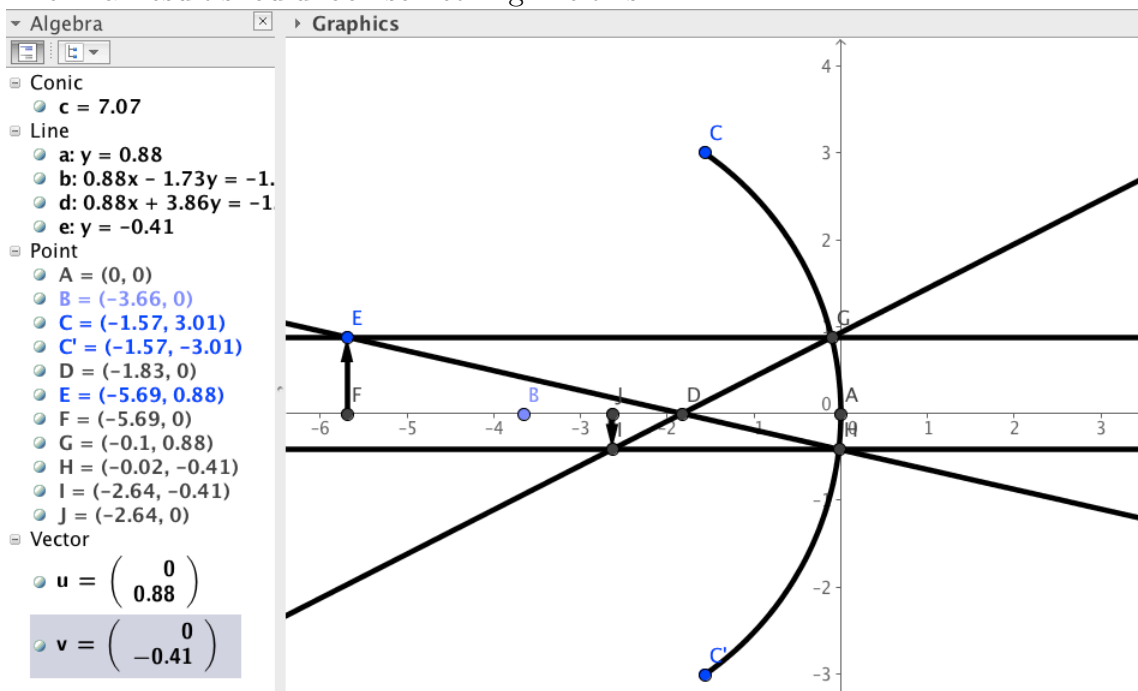
You can drag both the center of curvature and the top of the object to test different scenarios.

- Notice that when the object crosses the focal point, the ray diagram breaks. This is because the other intersection of the line with the mirror comes into play and we didn't draw a parallel line through that point. Do that and create a new image arrow.

2.2.3 Cleaning Things Up

This diagram works, but feels rather cluttered. We can do a few things to help.

- Hide points A, C', F, G, H, I, J, K, M, and L by clicking their blue radio buttons in the algebra view. Notice that the points can't be hidden when they are undefined but that once hidden, they stay hidden as the switch back and forth from defined to undefined. Likewise if you show them.

- To hide names of points, or to rename them, ctrl-click on the point either in the Algebra or the Graphics view and choose the appropriate option.

- Other improvements include turning Lines into Rays when we know where they start but not where they end or into Segments when we know both beginning and ending.

- Finally, we can color-code the rays and other things. Ctrl-click on an object (either in the Algebra or the Graphics view) and choose Object Properties. Using the various tabs, you can change colors and line thicknesses among other things.

- Hide the y-axis by ctrl-clicking on a blank space in the Graphics window and then selecting Graphics. (the last option). Select the y-axis tab.

- Use an actual image instead of the arrow. Choose the Image tool from the insertions collections. Click in the Graphics window and use the resulting dialog box to navigate to a suitable

image file on your computer. Then ctrl-click the image, choose Object Properties, choose the Position tab. Set corner 1 to be F (the base of the object vector) and corner 4 to be E (the top of the object vector). Execute similar steps for both image vectors. You can hide the vectors.
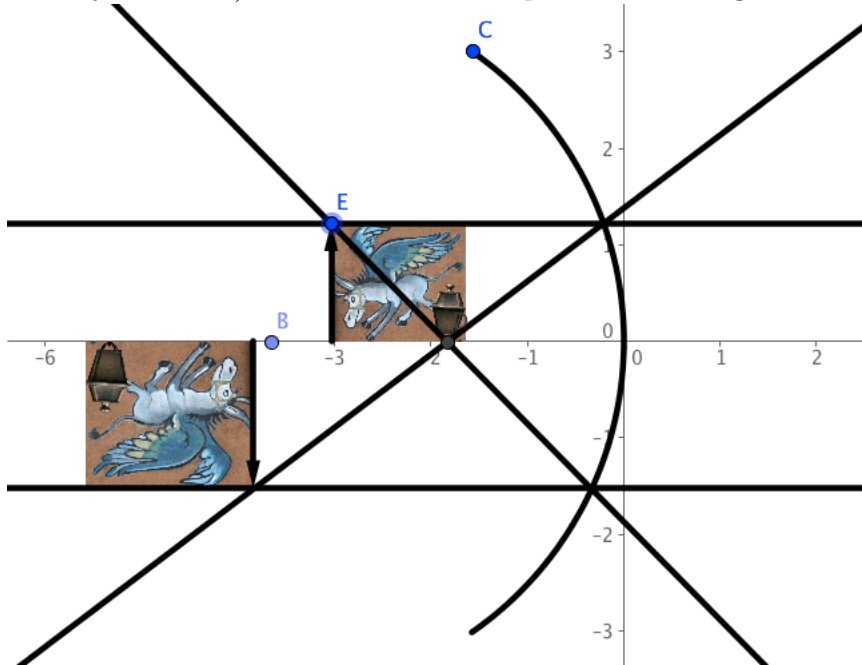
2.2.4 Learning from the Ray Diagram

The object can be moved around to show what locations result in real/imaginary images etc.

Notice that if they object is too tall relative to the size of the mirror and you have more than two rays that the rays don't all intersect at a single point. This is a real problem with spherical mirrors not an artifact of our rays or GeoGebra. Try replacing the mirror with a parabola (challenging in this orientation).

2.3 Projectile Motion

2.3.1 Useful Tools

- point ⬚ - A point placed in empty space will be moveable. These points are (usually) bright blue. A point placed on an axis, line, function, or conic section will move only along that axis, line,.... These points are (usually) light blue. Points which cannot be moved usually default to dark gray.

- slider ⬚ - Create an adjustable value. Can be integer or real. One slider at a time can be animated (i.e. told to automatically run through its entire set of values). Automation speed can be controlled as can whether it simply runs to completion, reverses, or jumps back to the beginning.

- input box ⬚ - Another way to create an adjustable value. No preset limits on the value. More precise than the slider. Note that input boxes are slightly harder to work with than may other objects because they don't show up in the Algebra view. They have to be ctrl-clicked in the Graphics view (and cannot be selected as a group there).

- command completion in `Input` bar - Start typing a command. After 3-5 letters a list of options will pop up. Use the up and down arrow keys to select the one you want. Hit return to put it into the Input bar for editing. The `<word>` pieces tell you what kind of information you need to add. If a command asks for a point, you can give it coordinates (cartesian as `(x,y)` or polar as `(r; angle-in-radians)`) or you can give it the name of a pre-existing point.

- up arrow in `Input` bar - scrolls back through the history of commands you have typed; very helpful when typing several similar commands

2.3.2 Working with Functions

Let's simulate throwing a rock from a variable location, at a variable angle, with a variable speed, on various planets.

- If the axes aren't showing, show them.
  - method 1: Click the icon under the window label "Graphics" that looks like axes. If the icon isn't visible, click the triangle next to the window label "Graphics".
  - method 2: Ctrl-click on the graphics window somewhere in the middle (not a border or the title bar) and select the axes icon in the resulting pop-up menu.
  - method 3: Click the gear-like icon in the group of 4 icons on the right side of the window and then choose graphics.

- Create the variables.
  - Create a movable launch point by using the Point tool and clicking anywhere but an axis in the Graphics view.
  - Create a way to set the launch angle by selecting the slider tool and then clicking somewhere out of the way in the Graphics view. (You can move sliders by ctrl-clicking them and then unchecking the Fixed option.) Select the angle option. Click Apply. If you like θ better then the default α as the name of your angle, then before you click Apply, click on the box at the right end of the name bar to pull up a symbol table and from that table select θ.
  - Create a way to set the launch speed.
    * In the Input bar type `v_0=2`. The value doesn't matter; we can change it later, but we need to create the speed variable.
    * Choose the Input Box tool and click somewhere out of the way in the Graphics window for the box to live. The caption is the label that will show. We want to link this input box to the variable v_0. Input boxes default to very long, so ctrl-click ON THE LABEL and select the style tab and then type a smaller number than 20 for the size of the box.
  - Create a slider or input box for the value of acceleration. Either make the acceleration a magnitude and put the minus sign for direction into your equation later (my habit) or make the acceleration negative and use a positive sign in the equation later.

- Describe the projectile's height as a function of its horizontal position (so the path is a function). Assuming the launch point is called A, the angle is α, and the gravitational acceleration is called g type
  `f(x)=y(A)+(x-x(A))*tan()-(g/2)*((x-x(A))/(v_0*sin()))^2` in the Input bar then put the cursor in each of the empty trig-function arguments in turn and click the box at the far right of

the input bar to select the angle. Hit return to enter the function. If you see red parentheses, that means your parentheses are unbalanced (too many openers or too many closers).

- This works (play with the parameters). However, we probably don't want the part of the parabola before the launch point. To get rid of it, double-click the function and add `if[x>=x(A),` to the beginning of the function and then `]` to the end.

- You should have something that looks roughly like this.
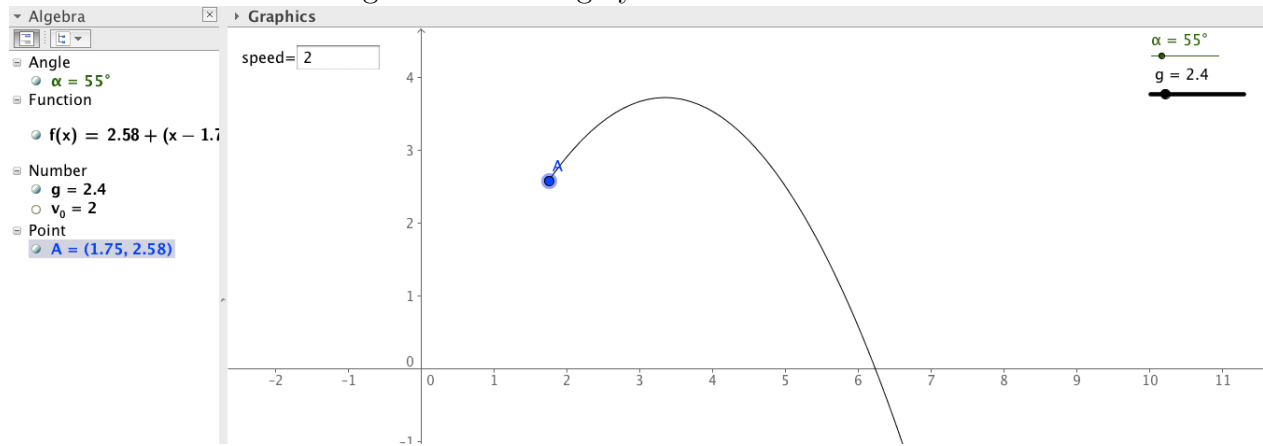
2.3.3   Working with Curves

The function version gives us the shape very nicely, but we've entirely lost the sense of time in the process. To recapture that, we need to work with parametric equations, called curves in GeoGebra.

You can build on top of the previous work - perhaps hiding the function - or use Save As and then delete the function but keep the controls.

- Create a time slider.

- Type `Curve[x(A)+v_0*cos( )*t,y(A)+v_0*sin( )*t-g/2*t^2,t,0,time]` in the Input bar and then hit return. Again, use the symbol table to insert the angle name where there are empty parentheses in the equation.

- Ctrl-click on the time slider and choose Animation On to trace the projectile's path.

This still leaves something to be desired because we can't see the roll of time afterwards so we can't point to it and discuss it afterwards very well. We have a couple of options. The first is easier to type (a little) but disappears every time we zoom or pan.

- In the Input bar type `(x(A)+v_0*cos( )*time,y(A)+v_0*sin( )*time-g/2*time^2)`. Actually, I used the up arrow key to get back the Curve typing I'd done and then modified that. Then ctrl-click on the point just created and select Trace On. Reset time to zero and click the triangle in the lower left of the Graphics view to run the animation. If you need to get rid of the dots, go to View - Refresh Views.

- Use the up-arrow key in the input bar to recall the point just entered and modify that input to `sequence[(x(A)+v_0*cos( )*t,y(A)+v_0*sin( )*t-g/2*t^2),t,0,time,0.1]`

Now we have something that clearly shows the distance traveled in a fixed time and that modifies nicely as we alter the various parameters.

2.3.4 Using the Spreadsheet

We can add drag to the problem (either linear or quadratic) without solving the relevant differential equations by using the spreadsheet to do iterative calculations.

- Create sliders for the size of the time step ($timeStep$), the drag coefficient ($drag$), and the power of velocity on which the drag depends ($power$).

- Show the spreadsheet view by selecting from the menus View -> Spreadsheet.

- Set up the conditions at $t = 0\,\text{s}$.

    - In cell A1 type 0 for the starting time.
    - In cell B1 type `=x(A)`.
    - In cell C1 type `=y(A)`.
    - In the View menu, select Keyboard. On the keyboard click the α to get the Greek keyboard. In cell D1 type `=v_0 cos( )` and use the keyboard to insert the angle symbol. Note that space works as a multiplication symbol.
    - In cell E1 type `=v_0 sin( )` and use the keyboard to insert the angle symbol.
    - In cell F1 type `=-sgn(D1)*drag*(abs(D1))^power`. The `-sgn(D1)` part makes the acceleration point the opposite direction of the velocity as a drag must.
    - In cell G1 type `-g - sgn(E1)*drag*abs(E1)^(power)`.

- Move one step forward in time.

    - In cell A2 type `=A1+timeStep`.
    - In cell B2 type `=B1+D1*timeStep`.
    - In cell C2 type `=C1+E1*timeStep`.
    - In cell D2 type `=D1+F1*timeStep`.
    - In cell E2 type `=E1+G1*timeStep`.
    - Select cells F1 and G1. Go to the lower right corner of the selection and drag down one row.

- To move forward further, highlight cells A2 through G2, go to the lower right corner of the selection and drag down 20 or 30 rows.

- Create the points to follow the motion.

    - In spreadsheet cell H1 type `=(B1,C1)`.
    - Select H1 and drag down to match the other columns.
    - With the entire column highlighted, ctrl-click on the column and uncheck Labels.

- You should have something that looks like this.

Algebra

- Angle
  - α = 55°
- Function
  - f(x) = 2.72 + (
- List
  - list1 = {(1.31, 2
- Number
  - A1 = 0
  - A2 = 0.1
  - A3 = 0.2
  - A4 = 0.3
  - A5 = 0.4
  - A6 = 0.5
  - A7 = 0.6
  - A8 = 0.7
  - A9 = 0.8
  - A10 = 0.9
  - A11 = 1
  - A12 = 1.1
  - A13 = 1.2
  - A14 = 1.3
  - A15 = 1.4
  - A16 = 1.5
  - A17 = 1.6
  - A18 = 1.7
  - A19 = 1.8
  - A20 = 1.9

Graphics

speed= 2

drag = 0.32

power = 1.5

timeStep = 0.1

α = 55°

g = 2.4

Spreadsheet

	A	B	C	D	E	F	G	H
1	0	1.31	2.72	1.15	1.64	−0.39	−3.07	(1.31…
2	0.1	1.43	2.88	1.11	1.33	−0.37	−2.89	(1.43…
3	0.2	1.54	3.02	1.07	1.04	−0.35	−2.74	(1.54…
4	0.3	1.65	3.12	1.04	0.77	−0.34	−2.62	(1.65…
5	0.4	1.75	3.2	1	0.51	−0.32	−2.52	(1.75…
6	0.5	1.85	3.25	0.97	0.25	−0.31	−2.44	(1.85…
7	0.6	1.95	3.27	0.94	0.01	−0.29	−2.4	(1.95…
8	0.7	2.04	3.27	0.91	−0.23	−0.28	−2.36	(2.04…
9	0.8	2.13	3.25	0.88	−0.47	−0.27	−2.3	(2.13…
10	0.9	2.22	3.2	0.86	−0.7	−0.25	−2.21	(2.22…
11	1	2.31	3.13	0.83	−0.92	−0.24	−2.12	(2.31…
12	1.1	2.39	3.04	0.81	−1.13	−0.23	−2.02	(2.39…
13	1.2	2.47	2.93	0.78	−1.33	−0.22	−1.91	(2.47…
14	1.3	2.55	2.8	0.76	−1.52	−0.21	−1.8	(2.55…
15	1.4	2.62	2.65	0.74	−1.7	−0.2	−1.69	(2.62…
16	1.5	2.7	2.47	0.72	−1.87	−0.2	−1.58	(2.7,…
17	1.6	2.77	2.29	0.7	−2.03	−0.19	−1.48	(2.77…
18	1.7	2.84	2.09	0.68	−2.18	−0.18	−1.37	(2.84…
19	1.8	2.91	1.87	0.66	−2.31	−0.17	−1.27	(2.91…
20	1.9	2.97	1.64	0.65	−2.44	−0.17	−1.18	(2.97…
21	2	3.04	1.39	0.63	−2.56	−0.16	−1.09	(3.04…
22	2.1	3.1	1.14	0.61	−2.67	−0.15	−1.01	(3.1,…