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Categorization and Representation of Physics Problems by Experts and Novices*

MICHELENE T. H. CHI

PAUL J. FELTOVICH

ROBERT GLASER

University of Pittsburgh

The representation of physics problems in relation to the organization of physics knowledge is investigated in experts and novices. Four experiments examine (a) the existence of problem categories as a basis for representation; (b) differences in the categories used by experts and novices; (c) differences in the knowledge associated with the categories; and (d) features in the problems that contribute to problem categorization and representation. Results from sorting tasks and protocols reveal that experts and novices begin their problem representations with specifiably different problem categories, and completion of the representations depends on the knowledge associated with the categories. For, the experts initially abstract physics principles to approach and solve a problem representation, whereas novices base their representation and approaches on the problem's literal features.

CATEGORIZATION AND REPRESENTATION OF PHYSICS PROBLEMS BY EXPERTS AND NOVICES

This paper presents studies designed to examine differences in the ways expert and novice problem solvers represent physics problems and to investigate implications of these differences for problem solution. A *problem representation* is a

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cognitive structure corresponding to a problem, constructed by a solver on the basis of his domain-related knowledge and its organization. A representation can take a variety of forms. Greeno (1977), for example, has proposed the representation of a problem as a constructed semantic network containing various components. Some of these correspond closely with the problem as stated, including the initial state (i.e., the "givens"), the desired goal, and the legal problem-solving operators (Newell & Simon, 1972). In addition, a representation can contain embellishments, inferences, and abstractions (Heller & Greeno, 1979). Since embellishment is one way of judging a solver's "understanding" of a problem (Greeno, 1977), it is possible that with increasing experience in a domain, the representation becomes more enriched. The research described here explores the changes in problem representation that emerge as a result of developing subject-matter expertise.

It is well known by now that the quality of a problem representation influences the ease with which a problem can be solved (Hayes & Simon, 1976; Newell & Simon, 1972). In physics, Simon and Simon (1978) have attributed the expert's "physical intuition" to the quality of the problem representation. The current consensus is that the expert's representation is superior because it contains a great deal of "qualitative" knowledge. De Kleer (1977), for example, has introduced both "quantitative" and "qualitative" components in the expert's representation of a physics problem where the qualitative component includes nonmathematical semantic descriptions of physical objects and their interactions. Novak's (1977) program ISSAC also suggested some characteristics of qualitative representation. In this program, physical objects from a problem statement are represented not literally, but rather, as abstract object categories—canonical object frames—each of which serves an equivalent physics role (e.g., pivot, lever, or point mass). The *canonical object frame* is a knowledge structure that augments the information about an object stated in a problem with associated information from the knowledge base. In his later work, Novak has proposed the inclusion of problem types in categorization by types (Novak & Araya, 1980). Categorization of a problem as a type would cue associated information in the knowledge base. Similarly, Reif (1979) has proposed a problem-solving model in which an initial step is a representation or "redescription of any problem in terms of concepts provided by the knowledge base" (p. 1). This knowledge base is arranged around "problem schemata," each of which contains information necessary to solve a specific category of problems.

The hypothesis guiding the present research is that the representation is constructed in the context of the knowledge available for a particular type of problem. The knowledge useful for a particular problem is indexed when a given physics problem is categorized as a specific type. Thus, expert-novice differences may be related to poorly formed, qualitatively different, or nonexistent categories in the novice representation. In general, this hypothesis is consistent with the "perceptual chunking" hypothesis for experts (e.g., Chase & Simon,

1973) and its more general cognitive ramifications (e.g., Chase & Chi, in press), which suggest that much of expert power lies in the expert's ability to quickly establish correspondence between externally presented events and internal models for these events.

More particularly, some evidence already exists in the literature to suggest that solvers represent problems by category and that these categories may direct problem solving. First, Hinsley, Hayes, and Simon (1978) found that college students can categorize algebra word problems into types, and that this categorization can occur very quickly, sometimes even after reading just the first phrase of the problem statement. For example, if subjects were to hear the words "a river steamer," then they might surmise that the problem was one about current, perhaps comparing the rates of going upstream and downstream. The ability to categorize problems quickly suggested to Hinsley et al. (1978) that "problem schemata" exist and can be viewed as interrelated sets of knowledge that unify superficially disparate problems by some underlying features. Secondly, in chess research, it appears that experts' superiority in memorizing chessboard positions arises from the existence of a large store of intact and well-organized chess configurations or patterns in memory (Chase & Simon, 1973). It is plausible that a choice among chess moves (analogous to physics solution methods) results from a direct association between move sequences and a configural (chunked) representation of the surface features of the board. Finally, from research in medical diagnosis, there is evidence to suggest that expert diagnosticians represent particular cases by general categories, and that these categories facilitate the formation of hypotheses during diagnosis (Pople, 1977; Wortman, 1972).

The accumulation of evidence for the importance of categorization in expert problem solving leads us to examine the role of categorization in expert physics problem solving: particularly, to investigate the relationships between such categorization and subsequent attempts at solution. The following series of studies attempts to determine: the categories that experts and novices impose on physics problems (Studies One and Two); the knowledge which these categorical representations activate in the problem solver (Study Three); and the cues or features of problems which subjects use to choose among alternative categories (Study Four).

Study One: Problem Sorting

The objective of the first study was to determine the kinds of categories subjects (of different experience) impose on problems. Using a sorting procedure, we asked eight advanced PhD students from the physics department (experts) and eight undergraduates (novices) who had just completed a semester of mechanics, to categorize 24 problems selected from Halliday and Resnick's (1974) *Fundamentals of Physics*, beginning with Chapter 5, Particle Dynamics, and ending with Chapter 12, Equilibrium of Bodies. Three problems were selected from

each chapter, and these were individually typed on 3 × 5 cards. Instructions were to sort the 24 problems into groups based on similarities of solution. The subjects were not allowed to use pencil and paper and, thus, could not actually solve the problems in order to sort them. As a test of consistency, subjects were asked to re-sort the problems after the first trial. Following this, they were asked to explain the reasons for their groupings. The time taken to sort on each trial was also measured.

Analysis of Gross Quantitative Results

No gross quantitative differences between the sorts produced by the two skill groups were observed. There were no differences in the number of categories produced by each group (8.4 for the experts and 8.6 for the novices), and the four largest categories produced by each subject captured the majority of the problems (80 percent for the experts and 74 percent for the novices). Likewise, experts and novices were equally able to achieve a stable sort within the two trials, that is, their second sort matched their first sort very closely. This suggests that their sorting pattern was not ad hoc, but rather, was based on some meaningful representation.

There were, however, some differences in the amount of time it took experts and novices to sort the problems. In fact, experts took longer (18 minutes or 45 seconds per problem, on the average) to sort the problems in the first trial than novices (12 minutes or 30 seconds). Both groups were relatively fast at sorting the second trial (4.6 minutes for the experts and 5.5 minutes for the novices). The speed with which the problems were sorted on the second trial (about 12 seconds per problem) suggests that subjects probably did not have to go through the entire process of "understanding" each problem again. Since the problems were all categorized after the first trial, the subjects probably needed only to identify the cues that elicited category membership.

In general, these quantitative data suggest that both experts and novices were able to categorize problems into groups in a meaningful way. Other than the difference in the time taken to sort on the first trial, there was little difference between skill groups. The critical question then becomes: what are the bases on which experts and novices categorize these problems?

Qualitative Analysis of the Categories

Analyses of Four Pairs of Problems. A cluster analysis (Diameter method) was performed on the problems grouped together by the experts and those by the novices. Such an analysis shows the degree to which subjects of each skill group agree that certain problems belong to the same group. One way to interpret the cluster analysis is to examine only those problems that were grouped together with the highest degree of agreement among subjects.

Our initial analysis centered on four pairs of problems. Figures 1 and 2 contain the diagrams of pairs of problems that were grouped together by the novices and the experts, respectively. These diagrams can be drawn to depict the physical situations described in the problem statements, and are sometimes given along with a problem statement (although no diagrams were given to the subjects in our studies). All eight novices grouped the top pair (Figure 1) together, and seven of the eight novices grouped the bottom pair. Both pairs of problems (Figure 2) were grouped together by six of the eight experts.

Examination of the novice pairs (Figure 1) reveals certain similarities in the surface structures of the problems. By "surface structures," we mean: (a) the objects referred to in the problem (e.g., a spring, an inclined plane); (b) the literal physics terms mentioned in the problem (e.g., friction, center of mass); or (c) the physical configuration described in the problem (i.e., relations among physical objects such as a block on an inclined plane). Each pair of problems in Figure 1 contains the same object components and configurations—circular disks in the upper pair, blocks on an inclined plane in the lower pair.

The suggestion that novices categorize by surface structure can be confirmed by examining subjects' verbal descriptions of their categories. (Samples are given in the figures.) Basically, according to their explanations, the top pair of problems involves "rotational things" and the bottom two problems "blocks on inclined planes."

To reiterate, the novices' use of surface features may involve either keywords given in the problem statement or abstracted visual configurations, that is, the presence of identical keywords (such as friction) is one criterion by which novices group problems as similar. Yet, novices were also capable of going beyond the word level to classify by types of physical objects. For example, "merry-go-round" and "rotating disk" are classified as the same object, as is the case for the top pair of problems in Figure 1.

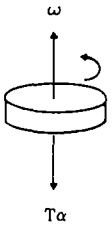
For experts, surface features do not seem to be the bases for categorization. There is neither great similarity in the keywords used in the problem statements, nor visual similarity apparent in the diagrams depictable from each pair of problems shown in Figure 2. Nor is the superficial appearance of the equations that can be used on these problems the same. Only a physicist can detect the similarity underlying the expert's categorization. It appears that the experts classify according to the major physics principle governing the solution of each problem. The top pair of problems in Figure 2 can be solved by applying the Law of Conservation of Energy; the bottom pair is better solved by applying Newton's Second Law (F = MA). The verbal justification of the expert subjects confirms this analysis. If "deep structure" is defined as the underlying physics law applicable to a problem; then, clearly, this deep structure is the basis by which experts group the problems.

Analysis of Categories. Further insight into the ways subjects categorize problems is given by the descriptions subjects gave for the categories they

Diagrams Depicted from Problems Categorized by Novices within the Same Groups

Novices' Explanations for Their Similarity Groupings

Problem 10 (11)

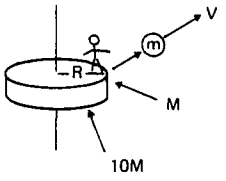


Novice 2: "*Angular* velocity, *momentum*, circular things"

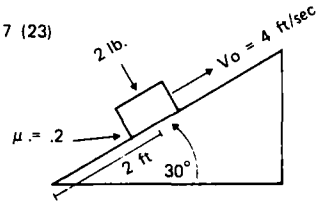
Novice 3: "*Rotational* kinematics, *angular* speeds, *angular* velocities"

Novice 6: "Problems that have something *rotating; angular* speed"

Problem 11 (39)



Problem 7 (23)



Novice 1: "These deal with blocks on an *incline plane*"

Novice 5: "*Inclined plane* problems, coefficient of *friction*"

Novice 6: "Blocks on *inclined planes* with angles"

Problem 7 (35)

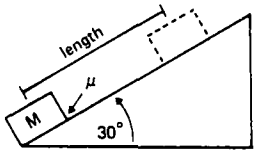
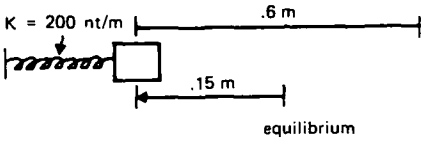


Figure 1. Diagrams depicted from two pairs of problems categorized by novices as similar and samples of three novices' explanations for their similarity are provided. Problem numbers given represent chapter, followed by problem number from Halliday and Resnick (1974).

Diagrams Depicted from Problems Catergorized by Experts within the Same Groups

Experts' Explanations for Their Similarity Groupings

Problem 6 (21)

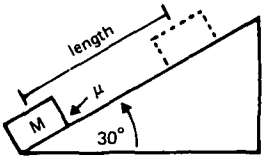


Expert 2: "*Conservation of Energy*"

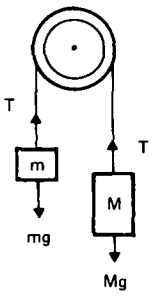
Expert 3: "*Work-Energy Theorem*. They are all straight-forward problems."

Expert 4: "These can be done from energy considerations. Either you should know the *Principle of Conservation of Energy*, or work is lost somewhere."

Problem 7 (35)



Problem 5 (39)



Expert 2: "These can be solved by *Newton's Second Law*"

Expert 3: "F = ma; *Newton's Second Law*"

Expert 4: "Largely use F = ma; *Newton's Second Law*"

Problem 12 (23)

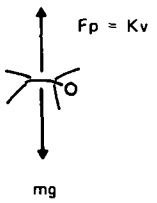


Figure 2. Diagrams depicted from pairs of problems categorized by experts as similar and samples of three experts' explanations for their similarity are provided. Problem numbers given represent chapter, followed by problem number from Halliday and Resnick (1974).

created. Tables 1 and 2 show the category descriptions (Column 1) used by more than one expert or novice. These category labels apply to all problems within each of their sorted piles.[1] Column 2 shows the number of subjects who used the category label. Column 3 shows the average size of the category among subjects who used it. And, Column 4 gives the total number of problems (out of 192, 24 problems for each of 8 subjects) according to category.

TABLE 1
Expert Categories

Category Labels	*Number of Subjects Using Category Labels* ($N_1 = 8$)	*Average Size of Category* ($N_2 = 24$)	*Number of Problems Accounted for* ($N_1 \times N_2$)
Second law	6	6.0	36
Energy principles (Conservation of Energy considerations, Work-Energy Theorem)†	6	5.5	33
*Momentum principles (Conservation of Momentum, Conservation of Linear Momentum, momentum considerations)†	6	5.0	30
*Angular motion (angular speed, rotational motion, rotational kinematics, rotational dynamics)†	6	3.0	18
Circular motion	5	1.6	8
*Center of mass (center of gravity)†	5	1.4	7
Statics	4	1.0	4
Conservation of Angular Momentum	2	1.5	3
*Work (work and kinetic energy, work and power)†	2	1.5	3
Linear kinematics (kinematics)†	2	1.5	3
Vectors	2	1.0	2
*Springs (spring and potential energy, spring and force)†	2	1.0	2

*Indicates the categories used by both novices and experts.

†When multiple descriptors across subjects were treated as equivalent, these are given in parentheses.

[1]For example, if a subject said of a problem group: "These all involve inclined planes, some with a frictional surface, some frictionless," the label "inclined planes" was counted since it applied to all problems in the set.

TABLE 2
Novice Categories

Category Labels	Number of Subjects Using Category Labels ($N_1 = 8$)	Average Size of Category ($N_2 = 24$)	Number of Problems Accounted for ($N_1 \times N_2$)
*Angular motion (angular velocity, angular momentum, angular quantities, angular speed)†	7	5.6	39
*Springs (spring equation, spring constant, spring force)†	6	2.8	17
Inclined planes (blocks on incline)†	4	3.8	15
Velocity and acceleration	2	5.5	11
Friction	2	5.0	10
Kinetic energy	4	2.0	8
*Center of mass (center of gravity)†	5	1.4	7
Cannot classify (do not know equations, do not go with anything else)†	4	1.8	7
Vertical motion	2	3.5	7
Pulleys	3	2.0	6
*Momentum principles (Conservation of Momentum)†	2	3.0	6
*Work (work, work plus Second Law, work and power)†	4	1.0	4
Free fall	2	1.0	2

*Indicates the categories used by both novices and experts.

†When multiple descriptors across subjects were treated as equivalent, these are given in parentheses.

There are several things to note about these data which confirm our initial analyses of the four pairs of problems. First, there is little overlap between expert and novice categories. Only five of 20 distinct categories (marked with asterisks) are shared by the two groups. Second, if one considers the four predominant categories (the upper four in the tables in each subject group, ranked by total number of problems in each), the only overlap is in the category "angular motion." In particular, for these predominant classifications, the novices' descriptions are mostly objects and other surface characteristics of problems, whereas descriptions given by experts all involve laws of physics. Third, although both experts and novices classify a large number of problems (61 percent

for the experts, 43 percent for the novices)[2] into four categories, there is a slight difference in the distribution of the problems across categories, which may suggest greater variability in novices' classification. Three major categories accounted for a sizable number (33 on the average) of experts' problems, whereas only one major category accounted for a large number (39) of novices' problems. This again suggests that experts are able to "see" the underlying similarities in a great number of problems, whereas the novices "see" a variety of problems that they consider to be dissimilar because the surface features are different.

Study Two: Sorting Problems with Surface Similarity

The objective of this study was to test our interpretations of Study One: experts categorize problems by laws of physics, and novices by surface features. A new

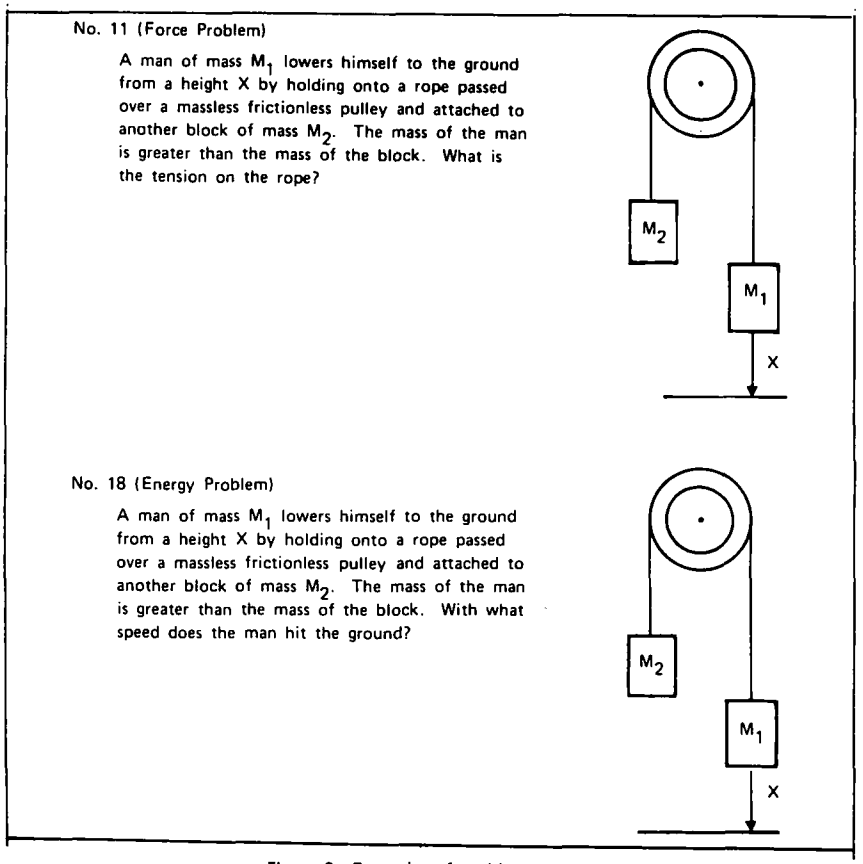


Figure 3. Examples of problem types.

[2]The percentages here do not correspond to those mentioned on page 5. Those were based on the largest sorting piles given by each subject, regardless of their contents or what they were labeled. Percentages here (Tables 1 and 2) are based on the sizes of specifically labeled categories when they were used by subjects.

set of 20 problems was constructed in which surface features were roughly crossed with applicable physics laws. Table 3 shows the problem numbers and the dimensions on which these problems were varied.[3] The left column indicates the major objects described in a problem. The three right headings are basic laws that can be used to solve problems. Figure 3 shows an example of a pair of problems that contain the same surface structure but different deep structure. In fact, they are identical, except for the question asked. Our prediction was that novices would group together problems that have the same surface structure, regardless of the deep structure, and experts would group together those problems with similar deep structures, regardless of the surface structure. Individuals of intermediate competence should exhibit some characteristics of each.

TABLE 3
Problem Categories

	Principles		
Surface Structure	*Forces*	*Energy*	*Momentum (Linear or Angular)*
Pulley with hanging blocks		20†	
	11	19†	
	14*	3*†	
Spring		7	
	18	16	1
			17+
		9	6+
Inclined plane	14*	3*†	
		5	
Rotational	15		2
			13
Single hanging block	12		
Block on block	8		
Collisions (Bullet-"block"			4
or Block-block)			6+
			10+

*Problems with more than one salient surface feature. Listed multiply by feature.
†Problems that could be solved using either of two principles, energy or force.
+Two-step problems, momentum plus energy.

[3]The problems were chosen from texts or constructed (to satisfy the a priori classification scheme) by Andrew Judkis, an assistant in the project who was a senior electrical engineering major with substantial experience in physics. Clearly, some problems could be solved using approaches based on either of two principles, Force and Energy, and in fact Judkis solved them both ways. In these cases, the problem is listed under the principle he judged to yield the simplest or most elegant solution but is marked with a cross. Also, some problems were two-step problems involving both momentum and energy. These are listed under the principle that seemed most important (in this case, momentum conservation) and are marked with a "+." These two-step problems are not designated explicitly as involving two principles. Some problems involve more than one potential physical configuration, e.g., "a pulley attached to an incline." These are marked with a single asterisk and listed multiply under alternative features.

The results confirm our previous interpretation. Table 4 shows the groupings and explanations of a novice who had completed one course in mechanics. This novice classification is based entirely on the surface structures of the problems. He collapsed problems across the physics laws, as was predicted. For example, the four problems in Group 2, 11 and 12 are force problems; 16 and 19, energy problems. The two problems in Group 4, classified by the novice as "Conservation of Energy," were problems purposely constructed as additional tests of "surface dependence" in novices; the novice identified them as energy problems only because they both have energy "cover stories" (i.e., they are stated in terms of energy), even though the major principle in each is Conservation of Momentum.

TABLE 4
Problem Categories and Explanations for Novice H. P.

Group 1:	2, 15	"Rotation"
Group 2:	11, 12, 16*, 19	"Always a block of some mass hanging down"
Group 3:	4, 10	"Velocity problems" (collisions)
Group 4:	13†, 17	"Conservation of Energy"
Group 5:	6, 7, 9, 18	"Spring"
Group 6:	3, 5, 14	"Inclined plane"
Groups 7, 8, 9 were singletons		

*Problem discrepant with our prior surface analysis as indicated in Table 3.
†Problems discrepant with our prior principles analysis as indicated in Table 3.

Table 5 shows the groupings of a physics graduate student. He classified the problems according to the three underlying physics laws specified a priori in Table 3. However, three of his classifications are discrepant with our analysis of the underlying principles. These discrepancies probably reflect deficiencies in his knowledge organization; the features in the problem statement cued the "wrong" category.

TABLE 5
Problem Categories and Explanations for Expert G. V.

Group 1:	3, 9, 2*, 17, 20, 5, 7, 19, 16	"Conservation of Energy"
Group 2:	13, 4, 10, 6, 15*, 1, 18*	"Conservation of Linear and Angular Momentum"
Group 3:	8, 12, 14, 11	"Statics problems or balance forces"

*Problems discrepant with our prior principles analysis.

That the graduate student's categorization was deficient is supported by Table 6 which shows the categories of a physics professor who sorted the problems after having spent considerable time thinking about how he would solve each problem in conjunction with a different task (reported in Study Four).

Hence, this subject's categorization can serve as a validation for our prior analysis of problem types (Table 3). Only one problem, 9, is sorted according to a principle different from our choice.

TABLE 6
Problem Categories and Explanations for Expert V. V.

Group 1:	2, 13	"Conservation of Angular Momentum"
Group 2:	18	"Newton's Third Law"
Group 3:	1, 4	"Conservation of Linear Momentum"
Group 4:	19, 5, 20, 16, 7	"Conservation of Energy"
Group 5:	12, 15, 9*, 11, 8, 3, 14	"Application of equations of motion" (F = MA)
Group 6:	6, 10, 17	"Two-step problems: Conservation of Linear Momentum plus an energy calculation of some sort"

*Problem discrepant with our prior principles analysis.

What would an individual of intermediate competence do? Table 7 shows the groupings of an advanced novice (a fourth-year undergraduate physics major). His representations of the problems are characterized by the underlying principles in an interesting way. These principles are qualified and constrained by the surface components included in the problems. For example, instead of classifying all the force problems together (Groups 4, 6, and 7), as did the expert, he explicitly separated them according to surface entities of the problems. However, although he did not strictly group problems by physics laws, neither did he uniformly group them according to surface features. For instance, Groups 3 and 6 were separated even though they both involved springs. In addition, his groupings of principle were substantially discrepant with our prior analysis and that of the physics professor (Expert V.V. Table 6).

TABLE 7
Problem Categories and Explanations for Advanced Novice M. H.

Group 1:	*14, 20*	"Pulley"
Group 2:	*1, 4, 6, 10, 12**	"Conservation of Momentum" (collision)
Group 3:	*9, 13*, 17, 18**	"Conservation of Energy" (springs)
Group 4:	*19, 11*	"Force problems which involve a massless pulley" (pulley)
Group 5:	*2, 15**	"Conservation of Angular Momentum" (rotation)
Group 6:	*7*, 16**	"Force problems that involve springs" (spring)
Group 7:	*8, 5*, 3*	"Force problems" (inclined plane)

Italic numbers mean that these problems share a similar surface feature, which is indicated in the parentheses, if the feature is not explicitly stated by the subject.

*Problems discrepant with our prior principles analysis.

To summarize the second study, we were able to replicate the initial finding that experts categorize physics problems by the underlying physics principles, a kind of "deep structure," whereas novices categorize problems by the surface structure of the problem. Furthermore, with learning, advanced novices begin to categorize problems by the principles with gradual release from dependence on the physical characteristics of the problems, although their groupings are still constrained by surface features.

Discussion of the Nature of the Representation

The results of the first two studies clearly indicate that the categories into which experts and novices sort problems are qualitatively different. However, neither group is classifying *solely* on the basis of the literal description of the problem statement. Both are able to read and gain some understanding of the problem, that is, to construct a somewhat enriched internal representation of it.

What is the relation between categorization and a subject's representation of problems? There are at least two plausible interpretations. One, that after the reading of a problem statement, a representation is formed, and based on that representation, the problem is categorized. The taxonomy of representations proposed by McDermott and Larkin (1978) offers a plausible interpretation for the present results. These authors have proposed that the problem solver progresses through four stages of representations as s/he solves a problem. The first stage is a literal representation of the problem statement (containing relevant keywords) and the fourth stage is the algebraic representation that results once equations are produced. The middle two are the most important. The second stage ("naive") representation contains the literal objects and their spatial relationships as stated in the problem and is often accompanied by a sketch of the situation (Larkin, 1980).[4] Such a representation and the accompanying sketch is "naive" because it can be formed by a person who is relatively ignorant of the domain of physics. The third stage ("scientific") representation contains the idealized objects and physical concepts, such as forces, momenta, and energies, which are necessary to generate the equations of the algebraic representation. This stage is related to the solution method. A plausible interpretation based on this framework is to postulate that novices' categorization is based on the construction of "naive" representations, with some limited elements of a "scientific" representation. Experts, on the other hand, may have constructed a more "scientific" representation, and based their categorizations on the similarities at this third level of representation. Such an interpretation would be consistent with the timing data of Study One: it could explain why experts actually took longer initially to classify the problems. They had to process the problems more

[4]In the McDermott and Larkin (1978) paper, they referred to the second stage of representation as the accompanying or produced diagram and to the third stage as the abstracted free body diagram. We took the liberty of corresponding the "naive" representation as the second stage and the "scientific" representation as the third stage, although Larkin (1980) has developed the ideas of "naive" and "scientific" representations beyond that of the diagram and the free body diagram.

"deeply" to a scientific representation in order to determine the principle underlying a problem.

An alternative interpretation for the nature of problem representation and its relation to categorization is to postulate more interaction among "stages" of representation than is proposed by McDermott and Larkin (1978). Under this interpretation, a problem can be at least tentatively categorized after some gross preliminary analyses of the problem features. After a potential category is activated, then the remainder of the representation is constructed for solution with the aid of available knowledge associated with the category. This interpretation is supported by the evidence that a problem can be categorized quickly (within 45 seconds, including reading time) and that it can often be tentatively categorized after reading just the first phrase of the problem (Hinsley et al., 1978; and our own results from Study Four). According to this interpretation, a problem representation is not *fully* constructed until after the initial categorization has occurred. The categorization processes can be accomplished by a set of rules that specify problem features and the corresponding categories that they should cue.

The second interpretation is our initial preferred hypothesis for the process of representing a problem for solution. It suggests that a problem representation is constructed in the context of the knowledge available for a problem type which constrains and guides the final form which the representation will take. A category and its associated knowledge within the knowledge base constitute a "schema," in Rumelhart's sense (in press), for a particular problem type. It is the content of these problem schemata (plural for *schema*) that ultimately determines the quality of the problem representation. Because the character of problem categories is different between experts and novices, we postulate that their problem schemata contain "different" knowledge. The next study presents a somewhat more direct look at the knowledge accessed by the category labels used by experts and novices.

Study Three: Contents of Schemata

We presume that the category descriptions provided by experts and novices (Tables 1 and 2) represent labels they use to access a related unit of knowledge, i.e., a schema. To assess the kind of knowledge that might be associated with these schemata, a selected set of 20 category labels, ranging from those generated predominantly by experts (e.g., Newton's Second Law, see Table 1) to those provided by novices (e.g., block on incline, see Table 2), was presented to two experts (M. G., M. S.) and two novices (H. P., P. D.). Subjects were given three minutes to tell everything they could about problems involving each category label and how these might be solved.

Analysis of Protocols as Node-link Structures

The protocols of one expert's (M. G.) and one novice's (H. P.) elaboration of the category label "inclined plane" can be grossly diagramed in the form of a node-link structure (Figure 4, 5). The network depiction shown in Figure 4

indicates that the novice's representation for "inclined plane" is very well developed. His representation contains numerous variables that can be instantiated, including the *angle* at which the *plane* is inclined with respect to the horizontal, whether there is a *block* resting on a plane, and the *mass* and *height* of the block. Other variables mentioned by the novice include the *surface property* of the plane, whether or not it has *friction*, and if it does, what the *coefficients of static and kinetic friction* are. The novice also discussed possible *forces* that may act on the block, such as possibly having a *pully* attached to it. The novice did not discuss any physics principles until the very end, where he mentioned the pertinence of Conservation of Energy. However, his mentioning of the Conservation of Energy principle was not elicited as an explicit solution procedure that is applicable to a configuration involving an inclined plane, as will be seen later in the case of the expert.

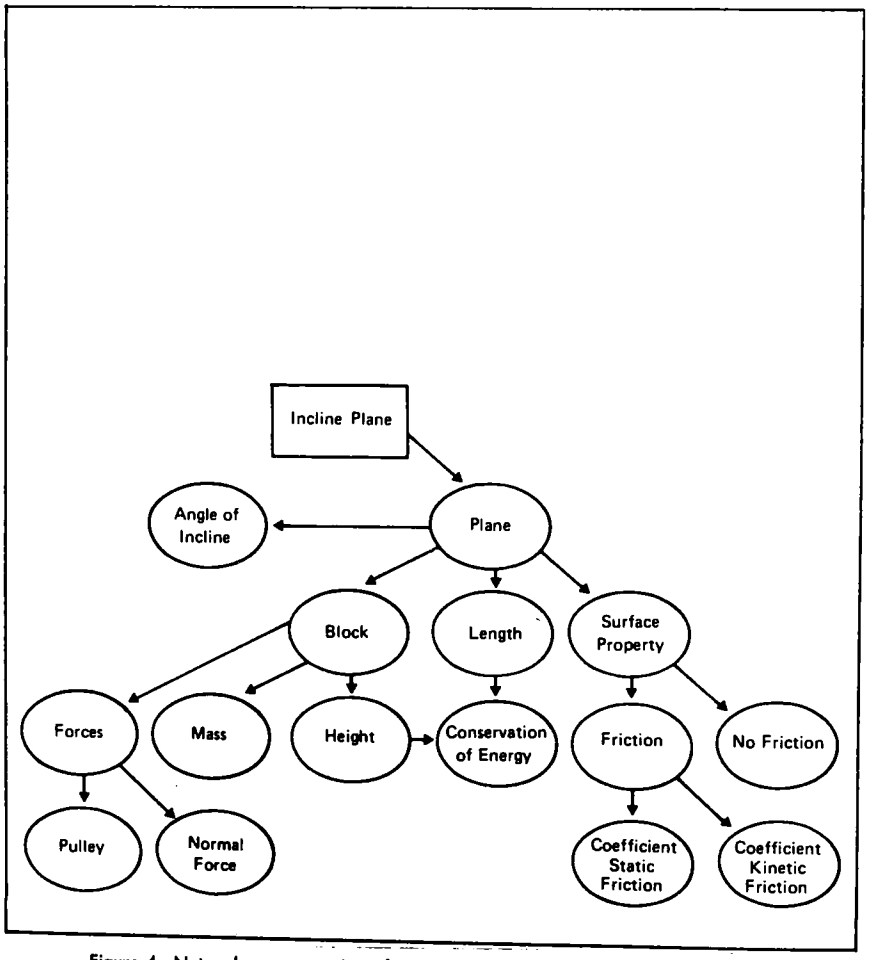


Figure 4. Network representation of Novice H. P.'s schema of an inclined plane.

The casual reference to the underlying physics principle given by the novice in the previous example is in marked contrast to the expert's protocol in which she immediately mentioned general alternative basic physics principles, Newton's laws of Force and Conservation of Energy, that may come into play for problems containing an inclined plane (Figure 5). The expert not only mentioned the alternative methods, but also the conditions under which they can be applied (dotted enclosures in Figure 5). Therefore, the expert appears to associate her principles with procedural knowledge about their applicability.

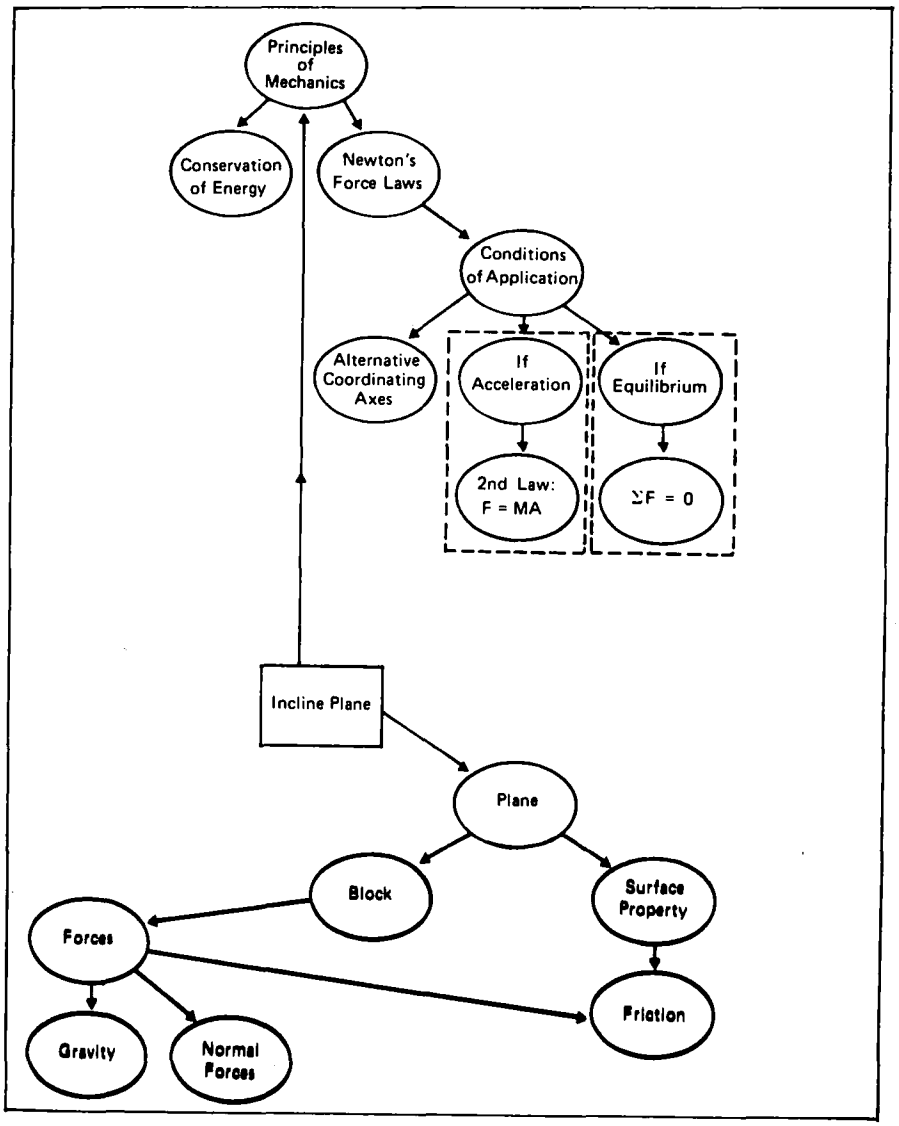


Figure 5. Network representation of Expert M. G.'s schema of an inclined plane.

After elaboration of the principles and the conditions of their applicability to inclined plane problems (depicted in the top half of Figure 5), Expert M. G. continued her protocol with descriptions of the structural or surface features of inclined plane problems (lower half of Figure 5), much like the description provided by Novice H. P. in Figure 4. Hence, it appears that this knowledge is common to subjects of both skill groups, but the expert has additional knowledge pertaining to solution procedures based on major physics laws.

Analysis of Protocols in the Form of Production Rules

An alternative way to analyze the same set of protocols is to convert them

TABLE 8
Expert Productions Converted from Protocols

M.S.

1. IF problem involves an inclined plane
   THEN a) expect something rolling or sliding up or down;
   b) use F = MA;
   c) use Newton's 3rd Law.
2. IF plane is smooth
   THEN use Conservation of Mechanical energy.
3. IF plane is not smooth
   THEN use work done by friction.
4. IF problem involves objects connected by string and one object being pulled by the other
   THEN consider string tension.
5. IF string is not taut
   THEN consider objects as independent.

M. G.

1. (IF problem involves inclined plane)*
   THEN a) use Newton's Law;
   b) draw force diagram.
2. (IF problem involves inclined plane)*
   THEN can use energy conservation.
3. IF there is something on plane
   THEN determine if there is friction.
4. IF there is friction
   THEN put it in diagram.
5. (IF drawing diagram)*
   THEN put in all forces—gravity, force up plane, friction, reaction force.
6. (IF all forces in diagram)*
   THEN write Newton's Laws.
7. IF equilibrium problem
   THEN a) $\Sigma F = 0$;
   b) decide on coordinate axes.
8. IF acceleration is involved
   THEN use F = MA.
9. IF "that's done" (drawing diagram, putting in forces, choosing axes)*
   THEN sum components of forces.

*Statements in parentheses were not said explicitly by the subjects but are indicated by the context.

TABLE 9
Novice Productions Converted from Protocols

H. P.

1. (IF problem involves inclined plane)*
   THEN find angle of incline with horizontal.
2. IF block resting on plane
   THEN a) find mass of block
   b) determine if plane is frictionless or not.
3. IF plane has friction
   THEN determine coefficients of static and kinetic friction.
4. IF there are any forces on the block
   THEN . . .
5. IF the block is at rest
   THEN . . .
6. IF the block has an initial speed
   THEN . . .
7. IF the plane is frictionless
   THEN the problem is simplified.
8. IF problem would involve Conservation of Energy and height of block, length of plane, height of plane are known
   THEN could solve for potential and kinetic energies.

P. D.

1. (IF problem involves an inclined plane)*
   THEN a) figure out what type of device is used;
   b) find what masses are given;
   c) find outside forces besides force coming from pulley.
2. IF pulley involved
   THEN try to neglect it.
3. IF trying to find coefficient of friction
   THEN slowly increase angle until block on it starts moving.
4. IF two frictionless inclined planes face each other and a ball is rolled from a height on one side
   THEN ball will roll to same height on other side.
5. IF something goes down frictionless surface
   THEN can find acceleration of gravity on the incline using trigonometry.
6. IF want to have collision
   THEN can use incline to accelerate one object.

*Statements in parentheses were not said explicitly by the subject but are indicated by the context.

directly into "production rules" (Newell, 1973). This can be done simply by converting all statements that can be interpreted as reflecting *if-then* or *if-when* structures in the protocols. This transformation is quite simple and straightforward and covers a majority of the protocol data. Tables 8 and 9 depict the same set of protocols as do Figures 4 and 5, except these also include the data of the other two subjects. Such an analysis captures differences between the expert and novice protocols in a more pronounced way, and other differences also become more apparent.

As suggested earlier, the experts' production rules (Table 8) contain explicit solution methods, such as "use F = MA," "sum all the forces to 0." These procedures may be considered as calls to action schemata (Greeno, 1980).

None of the novices' rules depicted in Table 9 contain any actions that are explicit solution procedures. Their actions can be characterized more as attempts to find specific unknowns, such as "find mass" (see rules H. P. 2 and P. D. 1 in Table 9). In addition, one novice (H. P.), exhibited a number of production rules that have no explicit actions. This suggests that he knew what problem cues are relevant, but did not know what to do with them, that is, if we think of the protocols as reflecting contents of an inclined plane schema, the novice's schema may contain fewer explicit procedures.

Finally, our network analyses (Figures 4 and 5) suggested that the mentioning of Conservation of Energy by Novice H. P. was somehow different from the mentioning of Conservation of Energy by the Expert M. G. This difference can now be further captured by this second mode of analysis. In Table 9, it can be seen that Novice H. P.'s statement of Conservation of Energy (Rule 8) was part of a description of the condition side of a production rule, whereas the statement of this principle by both experts (Table 8, see rules M.S. 2 and M.G. 2) is described on the action side of the production rules—supporting our previous interpretation of a difference in the way "Conservation of Energy" was meant when mentioned in the protocols of Novice H. P. (Figure 4) and Expert M. G. (Figure 5).

Study Four: Feature Identification

We have now claimed: (a) that experts and novices categorize problems differently; (b) that these categories elicit a knowledge structure (a schema) that functions in the representation of a problem; and (c) that at least for experts this schema includes potential solution methods. In this study, we attempt to determine problem features that subjects use in eliciting their category schema and, hence, their solution methods.

Subjects were asked to read problem statements and to think out loud about the "basic approach" that they would take toward solving the problem. Subjects were encouraged to report all thoughts and hunches they had while deciding upon a "basic approach," even if these ideas occurred during the reading of the problem. Following this unconstrained thinking period for each problem, subjects were asked to state their "basic approach" explicitly and to state the problem features that led them to their choice.

The subjects were two physicists who had frequently taught introductory mechanics and two novices who had completed a basic college course in mechanics with an A grade. The 20 problems used in this task were the same (described in Table 3) used for the sorting replication (Study Two); they have surface configurations crossed with principles.

Analyses of "Basic Approaches"

Table 10 gives the final "basic approaches" for all 20 problems, as stated by the two experts. Two aspects of these results are noteworthy. First, "basic ap-

proaches'' are interpreted by the experts as the major principles they would apply to solve the problems. In particular, these experts used the same terms for describing the basic solution method they would use as other experts have given in the sorting tasks. This task elicited responses consisting of the three major principles even more consistently than did the sorting task (compare Tables 10 and 1). For only one problem (Problem 1), did each expert use another term (center of mass). Second, intersubject agreement is nearly perfect. Only three problems (3, 5, 7) seemed like disagreements between the subjects. These arise from Expert J. L.'s use of ''work'' and Expert V. V.'s use of ''Conservation of Energy.'' Postexperimental discussion revealed that Expert J. L. distinguished between ''energy'' problems in which a dissipative force must be accounted for in the energy equation (work) and problems involving no dissipative force (strict Conservation of Mechanical Energy). Expert V. V. made no overt distinction between these types, treating the ''work'' problems as a special case of Energy Conservation.

Results from the two novices were impossible to analyze in the same way because these subjects were unable to produce any but the most general kind of

TABLE 10
Final Stated "Basic Approaches" of Experts V. V. and J. L.

	V. V.	*J. L.*
Problem 1	Center of mass	Center of mass
Problem 2	Conservation of angular momentum	Conservation of angular momentum
Problem 3	F = MA	Dynamics: F = MA or work
Problem 4	Conservation of momentum	Conservation of momentum
Problem 5	Conservation of energy	Dynamics: work
Problem 6	Conservation of momentum and conservation of energy	Conservation of energy
Problem 7	Conservation of energy	Work and energy
Problem 8	F = MA	F = MA
Problem 9	Conservation of energy or F = MA (favored) (not sure)	Conservation of energy
Problem 10	Conservation of momentum and conservation of energy	Conservation of momentum and conservation of energy
Problem 11	F = MA	F = MA
Problem 12	F = MA	F = MA
Problem 13	Conservation of rotational momentum	Conservation of rotational momentum (changed mind from conservation of energy)
Problem 14	F = MA	F = MA
Problem 15	F = MA	Pseudo F = MA
Problem 16	Conservation of energy	Conservation of energy
Problem 17	Conservation of momentum and conservation of energy	Conservation of momentum and conservation of energy
Problem 18	Newton's Third	Newton's Third
Problem 19	Conservation of energy	Conservation of energy
Problem 20	Conservation of energy	Conservation of energy

abstracted solution methods. In particular, when asked to develop and state "a basic approach," they did one of two things. They either made very global statements about how to proceed, "First, I figured out what was happening . . . then I, I started seeing how these different things were related to each other . . . I think of formulas that give their relationships and then . . . I keep on relating things through this chain . . .," or they would attempt to solve the problem, giving the detailed equation sets they would use.

Features Cuing the Principles

We examined the second portion of the protocols where subjects explicitly stated the features of the problems that led to their "basic approach." This analysis reveals several interesting aspects that are consistent with our interpretations from earlier experiments. Table 11 shows the frequency with which problem features were cited by the two experts and two novices as salient for leading to their "basic approach." A feature was included if it was mentioned at least twice (across 20 problems) by either of the two subjects, or once by both. The numbers given represent the number of problems for which each subject listed each feature as influential in his or her "basic approach" decision.

First, as can be seen in the table, the kinds of features mentioned as relevant by the novices are different from those identified as relevant by the experts. There is essentially no overlap in the features mentioned by novices and experts except for the object "spring." Relevant features selected by the novices are again literal objects and terms that can be identified in the problem statement, such as "friction," "gravity," etc. Features identified by the experts can be characterized as descriptions of the states and conditions of the physical situation described by the problem. In some instances, these are transformed or derived features, such as a "before and after situation" or "no external forces." Because these features are not explicitly stated in the problem, we refer to these features as second-order features. Second-order features are almost never mentioned by the novices.

Since second-order features must necessarily be derived from more literal surface features that are in the problem statements, it is of interest to see whether the surface features in the problem statement that elicit these second-order features can be identified. In order to do this, we can examine the initial part of the protocols (deciding the "basic approach") where second-order features were mentioned and infer the literal surface features from which these were elicited. Such inferences can be made more easily from protocols in which subjects gave responses after reading segments of the problem statement. In such cases, we can make mappings between what was read and what was said. In any case, such inferences are difficult; they must be speculative.

Table 12 categorizes the "basic approaches" given by Expert V. V. into three main principles shown in Column 1. Column 2 lists second-order features

TABLE 11
Key Features Cited by Experts and Novices

	Experts	
	V. V.	*J. L.*
Given initial conditions	9	3
Before and after situations	3	4
Spring	0	5
No external force	4	1
Don't need details of motion	4	1
Given final conditions	5	0
Asked something at an instant in time	4	1
Asked some characteristics of final condition	4	0
Interacting objects	0	4
Speed–distance relation	0	4
Inelastic collision	2	2
No initial conditions	4	0
No final conditions	4	0
Energy easy to calculate at two points	1	2
No friction or dissipation	3	1
Force too complicated	0	3
Momentum easy to calculate at two points	2	1
Compare initial and final conditions	2	0
Can compute work done by external force	2	0
Given distance	1	1
Rotational component	0	2
Energy yields direct relation	0	2
No before and after	2	0
Asked about force	2	0
	Novices	
	P. D.	*J. W.*
Friction	3	5
Gravity	3	3
Pulley	3	3
Inclined plane	3	2
Spring	2	3
Given masses	3	2
Coin on turntable	1	1
Given forces	1	1
Force–velocity relation	0	2

he often identified as helpful in deciding on a "basic approach." Column 3 gives examples of "surface" information from problem statements that we infer contributed to Expert V. V.'s second-order features. For example, it appears that Expert V. V. judges a problem to be a Conservation of Momentum problem when it involves a "before and after" situation with "no external forces or torques." "Before and after" situations, in turn, are identified in a problem when it has either a physical process with end points (e.g., something starts and

eventually stops) or a physical state that changes abruptly (e.g., there is a point where the girl has the rock and a point after which she does not). "No external forces" can sometimes be directly derived from the given problem, such as "neglecting friction" or may involve complex inference on the subject's part. Clearly, for the expert, even "first-order" features that feed second-order features can themselves be complex interactive information.

TABLE 12
The First- and Second-order Features that Elicited Expert V. V.'s Final "Basic Approach"

Principles	*Second-order Features (Derived Features)*	*First-order Features (Surface Features)*
Conservation of Momentum (Problems 2, 4, 13)	Before and after situation	Girl on *still* merry-go-round *throws* a rock . . . Two initially separated wheels are *suddenly coupled*.
	No external forces	Neglecting friction.
		No third entity mentioned except the interacting wheels.
Conservation of energy (Problems 5, 7, 16, 19, 20)	Before and after situation	Block *dropped* from a height X *onto* a spring. Block *starts* with initial velocity V. How *far* will it slide?
	Given or well-defined initial conditions	Initial height = X. Initial velocity = 0. Initial velocity = V.
Force Laws (Problems 3, 8, 9, 11, 12, 14, 15)	Determination of something at an instant in time	Break point of a rope. Coin observed to slide at distance R from center of turntable.
		Raising point of a disk.

Even though the experts cite the abstracted features as the relevant cues (Table 11), the basic keywords utilized from the problem by the two groups may still be the same, as was suggested from the results of Study Three. A direct way to ascertain whether subjects of different skills consider the same set of words as important, is to ask them to circle those words in the problem statements. In a separate study, eight novices and eight experts (graduate students) were asked to circle those words in the (previously used) 20 problems that they thought were relevant in helping them decide how difficult a problem is to solve. Although the task—requesting sources of difficulty—is slightly different from those used in Study Four, the results show a large overlap in the keywords selected by both groups. The only difference is that experts tended to identify fewer sets of words as relevant to their judgments as compared to the novices. However, almost always, the keywords chosen by the experts are subsets of those chosen by the novices.

Analysis of the Process of Problem Representation

Throughout this paper, our working definition of a problem representation has been that it is an internal cognitive structure constructed by a problem solver to

"stand for" or model a problem. In our discussion at the end of Study Two, we speculated that for both experts and novices, a problem representation is constructed within the constraints of the category knowledge (schema) that the problem activates. Hence, the resulting problem representation is an outcome of both the initial categorization process (resulting from an analyses of cues in a bottom-up manner) and the completion of a representation based on the knowledge available (top-down processing). We are now able to investigate this interactive process of categorizing and representing a problem more directly, by examining the subjects' protocols as they decide on a "basic approach."

In general, early in the reading of a problem, the expert usually entertains a hypothesis, a potential physics principle, or a set of plausible competing hypotheses. (Expert J. L., for example, generated her first principle(s) after reading 20 percent of each problem on the average.) This is followed by the extraction from the problem of additional features, which are used to confirm, reject, or choose among hypothesized principles.

A process of this kind is shown in Figure 6, which gives a schematic analysis of Expert J. L.'s development of a "basic approach" for Problem 16. Problem segments (Column 1) and protocol segments (Column 4) represent actual subject break points in the reading of the problem; that is, after having read the phrase *A block of mass M is dropped from a height X,* Expert J. L. paused and gave the protocol indicated in Column 4. Columns 2 and 3 represent our analysis of the possible second-order features and principles that the subject is deriving from that particular segment of the problem. Our interpretation is based on both the contents of her protocol at that point in time and her comments during the later probing section of the interview when she explicitly mentioned the features (see Table 11) that lead her to a final "basic approach."

In following the trace of Expert J. L.'s protocol given in Figure 6, we hypothesize that literal elements ("dropped" and "height X") directly trigger, for the expert, the possibility that the problem is one involving Conservation of Energy. Activating the Conservation of Energy schema, in turn, generates "slots" that guide the further interrogation of the problem. We interpret these slots to include, for example, the specification of "well-defined final conditions" and hence, a "before and after situation." In contrast to the slots of the novice (to be discussed later), the slots generated by this expert are at a high level. That is, filling these slots requires transformation of the literal features in the problem statement.

In particular, in the protocol, as slots are filled with first- and second-order problem features, the hypothesized Conservation of Energy representation is maintained. Final consideration of "maximum spring compression" completes the requirement for a "before and after" second-order feature, which, in turn, along with "no dissipative forces," confirms the validity of the Conservation of Energy schema as a representation for the problem.

It is also quite clear from Expert J. L.'s final comments in Figure 6 that the process of "instantiating" the Conservation of Energy schema for the problem

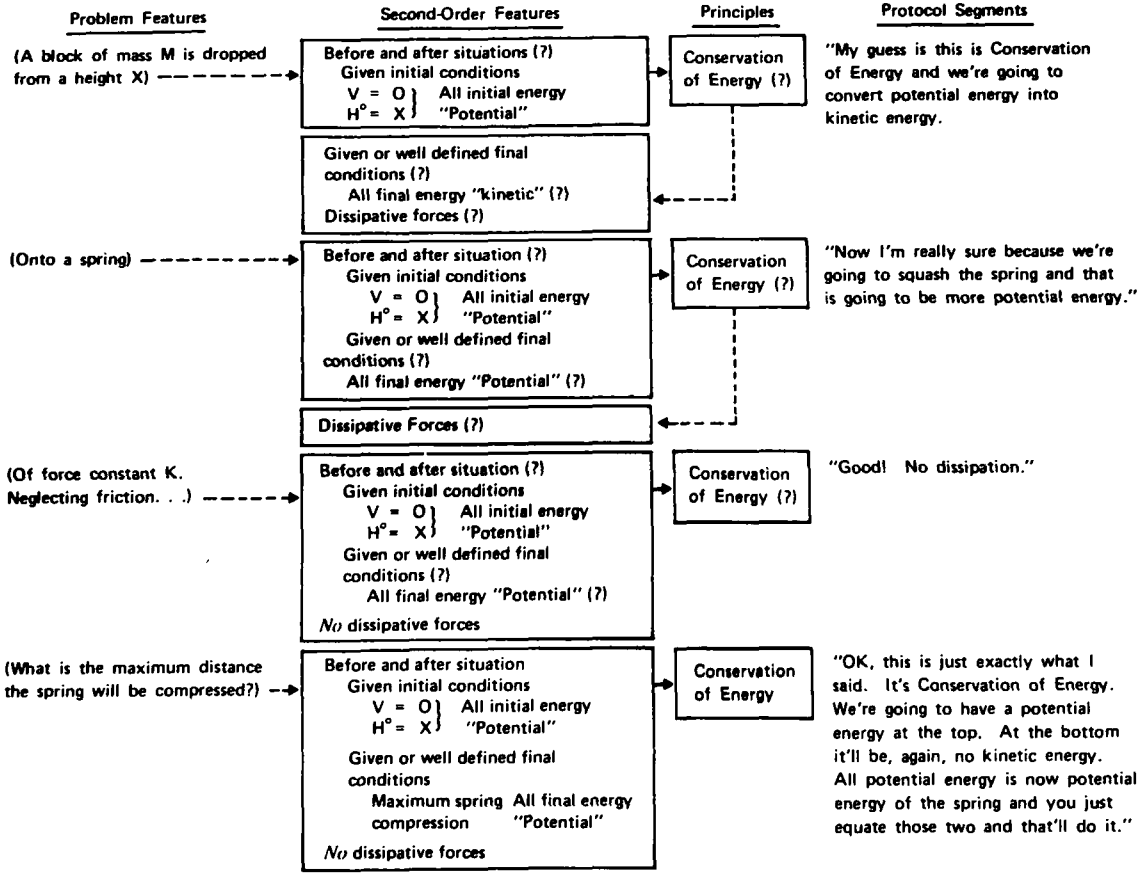


Figure 6. An example of Expert J. L.'s development of a "basic approach" during reading of a problem. "(?)" indicates hypotheses yet to be confirmed.

has yielded a general, abstract form for the equation that will be used in solution, that is, equating the "potential at the top" with the "potential energy of the spring" at the bottom. We presume that in full solution of the problem, the subject would proceed to represent these terms symbolically and to manipulate the resulting equation algebraically.

The protocol of a novice on the "basic approach" task for the same problem is given in Figure 7. Because the subject gave no protocol before reading the entire problem, we created hypothetical problem segments (Column 1) based upon our interpretation of his protocol. Column 2 is comprised of equations that can be derived from his protocol in Column 3. In this example, we presume that the idea of falling as indicated by *A block of mass M dropped from a height X* elicits the idea of gravity which, with the addition of a mass, generates the equation F = Mg. The "spring" and the "spring constant" suggest the equation F = −kx. Following the generation of these two separate and parallel knowledge states, the novice sees a common element between them which is "F," the forces. This enables him to equate the two, thereby eliminating the unknown.

Our interpretation for the novice in this case is that his problem representation is guided by two surface-oriented schemata, one associated with "springs" and the other with "gravity." The "slots" for these schemata would be related to the variables of a problem, comparable to those generated for an "inclined plane" schema shown in Figure 4. The process of representation is concerned mostly with finding the values of these variables, through equations that relate them.

Summary. Three kinds of analyses were carried out in Study Four: (a) an analysis of the basic solution methods ("basic approach") that subjects apply to problems; (b) the identification of features in the problem statement leading to a "basic approach"; and (c) an analysis of the process of constructing a problem representation.

The analyses of "basic approaches" provided results consistent with those found in the first two studies. In particular, experts gave "basic approaches" defined by the major physics principle they would apply to the problem, and there was near total agreement between the experts regarding the principle they would apply. The results for experts further established the relationship between problem categorization, as exhibited by experts in Studies One and Two, and methods of solution; experts categorize problems according to abstracted solution procedures. Novices were unable to formulate solution methods at intermediate levels of abstraction between "meta-level" prescriptions for how to proceed and highly specific equations. This suggests that novices' surface-oriented categorizations yield equations associated with these surface components.

The analysis of the features suggests that experts perceive more in a problem statement than do novices. They have a great deal of tacit knowledge that can

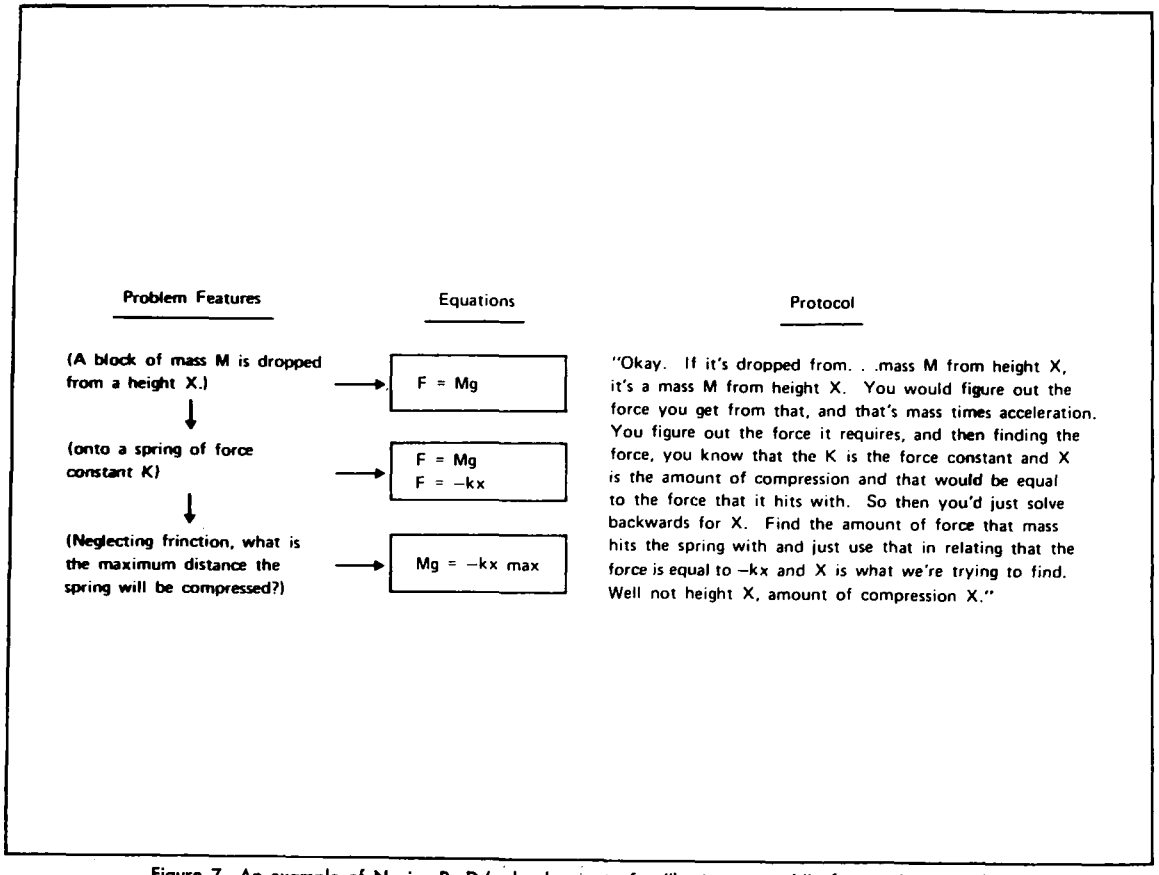


Figure 7. An example of Novice P. D.'s development of a "basic approach" after reading a problem.

be used to make inferences and derivations from the situation described by the problem statement. Their selection of the "principle" to apply to a problem seems to be guided by this second-order, derived knowledge. Hence, even though the same set of keywords may be deemed important by subjects of both skill groups, the actual cues used by the experts are not the words themselves, but what they signify. Novice features eliciting what they considered to be a "basic approach" were, again, literal problem components leading to equations.

The final analysis in Study Four investigated the process of constructing a problem representation. For experts, it was suggested that this process occurs over a span of time and involves interplay between the problem statement and the knowledge base—even during the reading of the problem. Literal cues from the problem statements are transformed into second-order (derived) features which activate a category schema for a problem type. This schema is organized by a physics law. It guides completion of the problem representation and yields a general form for the equations to be used in problem solution. For novices, problem representation is organized by schemata for object categories, for example, "spring problems" or "falling bodies." These yield equations specific to problems at these levels, and much of the process of problem representation involves instantiating the variables in these equations.

GENERAL DISCUSSION

Our research goal has been to ultimately understand the difference between experts and novices in solving physics problems. A general difference often found in the literature (Larkin, McDermott, Simon, & Simon, 1980; Simon & Simon, 1978) and also in our own study (Study Four, examining the processes of arriving at a "basic approach") is that experts engage in qualitative analysis of the problem prior to working with the appropriate equations. We speculate that this method of solution for the experts occurs because the early phase of problem solving (the qualitative analysis) involves the activation and confirmation of an appropriate principle-oriented knowledge structure, a schema. The initial activation of this schema can occur as a data-driven response to some fragmentary cue in the problem. Once activated, the schema itself specifies further (schema-driven) tests for its appropriateness (Bobrow & Norman, 1975). When the schema is confirmed, that is, the expert has decided that a particular principle is appropriate, the knowledge contained in the schema provides the general form that specific equations to be used for solution will take. For example, once the problem solver has decided to use an energy conservation approach, the general form of the solution equation involves energy terms equated at two points. The solver then needs only to specify these terms for the problem at hand. Such initial qualitative analysis would naturally lead to a more forward-working character (Larkin et al., 1980) of problem solving for the expert, in that the equations used

depend more on the way the problem is represented than on the "unknown." Although the problem unknown obviously cannot be ignored by the experts, the status of the unknown in the expert solution method appears secondary to that of deciding which physics principles have their conditions of applicability met in the problem. Hence, analogous to the way that a chess expert's initial classification yields a small set of "good" alternative moves, which must then be investigated analytically (Chase & Simon, 1973), the physics expert's initial categorization restricts search for a particular solution to a small range of possible operations.

Consistent with this point of view, the exploratory studies reported here suggest that problem solving in a rich knowledge domain begins with a brief analysis of the problem statement to categorize the problem. The first two studies showed that experts tended to categorize problems into types that are defined by the major physics principles that will be used in solution, whereas novices tend to categorize them into types as defined by the entities contained in the problem statement. We view the categories of problems as representing internal schemata, with the category names as accessing labels for the appropriate schemata. Although it is conceivable that the categories constructed by the novices do not correspond to existing internal schemata, but rather represent only problem discriminations that are created on the spot during the sorting tasks, the persistence of the appearance of similar category labels across a variety of tasks gives some credibility to the reality of the novices' categories even if they are strictly entities related.

Since our conception of problem solving is that it begins with the typing of the problem (or activating the appropriate schema) in a bottom-up manner by analyzing the problem features, Study Four attempted to capture these features. It appears that both skill groups use the same basic set of features in the problem statement, but the cues themselves and their interactions engage greater tacit knowledge for the experts than the novices. Experts then base their selection of the appropriate principle on the resulting second-order, derived cues. Novices basically use the features explicitly stated in the problem.

Furthermore, we presume that once the correct schema is activated, knowledge—both procedural and declarative—contained in the schema is used to further process the problem in a more or less top-down manner. The declarative knowledge contained in the schema generates potential problem configurations and conditions of applicability for procedures which are then tested with what is presented in the problem statement. The procedural knowledge in the schema generates potential solution methods that can be used on the problem. This type of interactive processing (both top-down and bottom-up) seems to be consistent with that captured in Study Four when subjects were simply asked to generate a "basic approach" to the problem.

In order to ascertain whether our initial hypothesis about the contents of the problem schemata is correct, Study Three attempted to assess their contents by

asking subjects to elaborate on them. Such initial analyses have begun to show clear differences between the problem schemata of experts and those of novices: Experts' schemata contain a great deal of procedural knowledge, with explicit conditions for applicability. Novices' schemata may be characterized as containing sufficiently elaborate declarative knowledge about the physical configurations of a potential problem, but lacking abstracted solution methods.

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