Nothing is Dark in the New Physics

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Abstract: *A geometrical explanation of Dark Matter and Dark Energy is easy to develop. It makes testable predictions and is thus completely falsifiable. The theory is based on a macroscopically extended fourth dimension of space, yielding a five-dimensional space-time structure. In this structure, the four-dimensional space-time of relativity is extrinsically curved in the higher spatial dimension. Dark Matter is curvature in the higher dimension that is not directly associated with local matter, but instead a result of the interaction between local matter or curvature and the global curvature due to all matter in the universe. However, criticisms have been leveled that the theory is not mathematical, i.e., there were no algebraic equations to describe the geometrical structure. Indeed, it seemed that no mathematical equations were even suggested by the geometry. However, a simple algebraic equation that describes and explains the geometry of the four-dimensional structure of space has since been derived by considering the particulate source of the DM halo. Although the algebraic equation appears to be Newtonian, it implies a five-dimensional unification of the gravitational and electromagnetic fields such as that accomplished by Kaluza in 1921 and extended by Einstein and his colleagues in the late 1930s. The new equation also shows how gravity can be quantized on the basis of relativity without hypothesizing the discrete nature of matter, i.e., without the existence of specific 'particles' of gravity as is inherent in quantum gravity theories, the Standard and other quantum models*.

Key Words: galactic rotation problem, halo, Dark Matter, CDM, Dark Energy, General Relativity, electromagnetism, Einstein, Kaluza, five-dimensional, fourth dimension, four-dimensional space, space-time, positive curvature, extrinsic curvature, compactification, unification

Introduction

In the past few decades, two 'crises' have emerged in the fields of cosmology and theoretical physics: The existence of Dark Matter (DM) and Dark Energy (DE). Both 'crises' require the immediate attention of the physics and scientific communities, yet both seem to completely defy any explanation based on the present paradigms of physics: Either the quantum or relativity. A large number of scientists have thus concluded that solving either or both of these 'crises' will lead to a new scientific revolution. Yet the scientific community as a whole has yet to come to grips with the revolutionary nature of these 'crises'. Although physics has always been about matter and the motion of matter, at least since Aristotle wrote his book *Physics* on the subject and established a course for later science to follow, science has never directly defined or understood the true nature of matter. Even now, scientists seem to attack these 'crises' separately, but will quickly admit that DM and DE must be related at some fundamental level. So the mere observed existence of DM and DE is finally forcing science to look directly at and seriously consider the fundamental nature of matter for the first time.

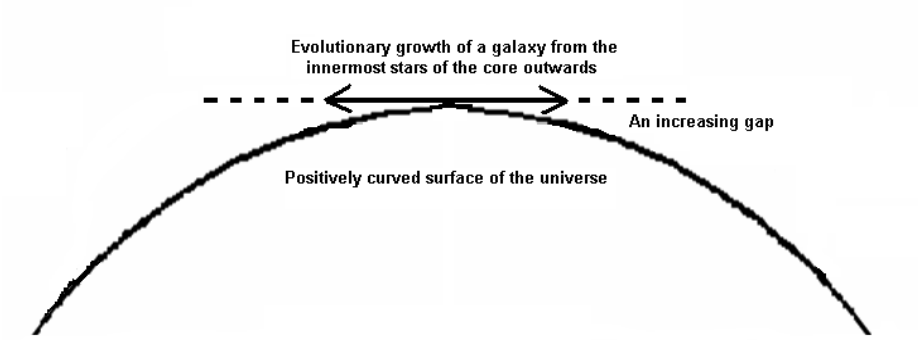
In the first case, it would seem that the DM halo that surrounds spiral galaxies and pumps extra gravitational energy into the stars in the galactic arms, thus yielding constant relative velocities, cannot justify the belief that the

haloes are filled with normal dim matter in the form of unidentified elementary particles or interstellar dust, black holes, brown dwarf stars and errant large planets. The halos seem to contain as much as ten times more matter than that contained in the galactic cores, which makes the existence of that much normal dim matter or as yet undiscovered elementary particles highly unlikely. On the other hand, the source of DE that seems to be propelling the expansion of the universe to ever greater speeds is even harder to pinpoint and speculation of its origins abound throughout science. Those speculations run from various forms of Modified Gravity (MG) theories, which either alter or seek to replace standard general relativity in some way, to having a variable gravitational constant G, altering Newton's Second Law of Motion (MOND or Modified Newtonian Dynamics), inventing and adding strange new elementary particles (such as accelerons) to the more common particle zoo, or developing a mysterious new non-material substance (quintessence) that fills all of the vastness of empty space. Furthermore, it is doubted that quantum theory can offer any real solutions to the 'crisis' because quantum theory predicts the existence of a zero point energy in the vacuum that is 10^{120} times stronger than the DE that permeates empty space. Any quantum theory of DE would need to explain this glaring discrepancy. Yet in spite of the great difficulty in explaining DM and DE that is indicated by all of these various speculations, one simple alteration to existing physics yields a complete geometrical solution to both 'crises' at the same time: The adoption of a real physical fourth dimension of space.

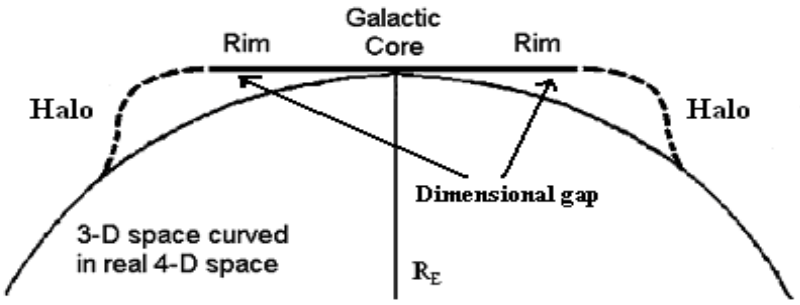
The geometrical model

The basic hypothesis of the new physics merely states that our common space of experience is not three-dimensional, but four-dimensional. Matter, normal gravity and the electric field are three-dimensional, but they are embedded in a four-dimensional space that is characterized by the common magnetic field, which is also four-dimensional. Such a four-dimensional structure is already implied by Maxwell's electromagnetic theory and was first investigated by William K. Clifford in the 1870s. In this more modern representation, the curvature of the space-time continuum, as described by general relativity, is 'extrinsic' instead of 'intrinsic' to four-dimensional space-time as normally interpreted. The 'extrinsic' nature of curvature would require a fifth-dimension of space-time that would be equivalent to the fourth dimension of space. This particular view of general relativity is completely compatible with Einstein's and other relativistic models and explains the five-dimensional unification of gravity and electromagnetism that Theodore Kaluza developed in 1921.

According to general relativity, our four-dimensional space-time continuum forms a Riemannian surface with positive curvature. The simplest such surface would be spherical or elliptical, but the overall shape of the surface is not relevant to this model as long as the curvature is positive. Given the reality of the hypothetical fourth dimension of space, a galaxy would grow from its center outwards by the accretion of material, but it would grow outward according to its own three-dimensional nature and the gravitational, electrical and magnetic forces that render it three-dimensional, and not the three-dimensionality of the normal space that is represented by the overall curvature of space-time. Under those circumstances, the galactic plane would grow tangent to the positively curved surface of the universe as a whole because the internal three-dimensional forces that distinguish the galaxy are far stronger than the local portion of the curvature resulting from the three-dimensionality of the universe as a whole.

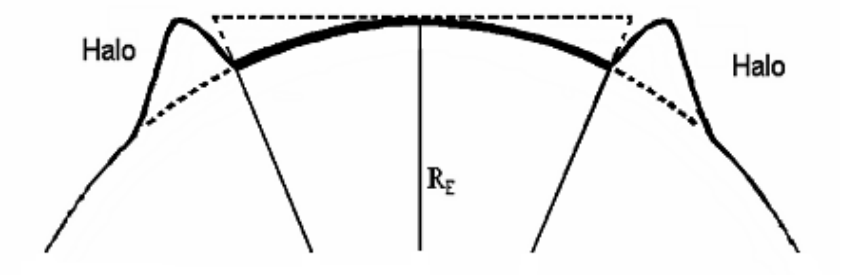


In other words, the galaxy grows beyond the curvature of the universal three-dimensional space and thus opens up a gap between its own internal three-dimensional plane and that of the curved surface of the universe.



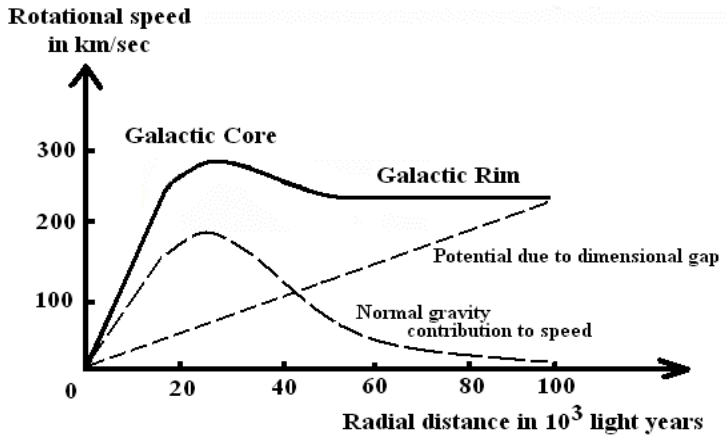
The quantity R_E is the Einstein radius and has a value between 10 billion and 13.8 billion light years. It determines the rate of curvature of space-time, but it also determines the weakness of the curve in that the surface curvature of the universe is so slight as to be negligible (very nearly but not quite Euclidean flat) on the local scale. However, the surface of the universe must also remain continuous with the outer stars in the galaxy and thus the surface of the positively curved universe would bend to meet the outer edge of the galactic plane, forming the halo.

Since we observe the universe as a whole through its own three-dimensionality, in that the electromagnetic waves by which we see other galaxies and celestial objects travel to us through three-dimensional space that follows the overall curvature of the universe, we would observe the galaxy 'as if' it existed completely within the three-dimensional curved surface of the universe.



Therefore, we 'observe' galaxies as part of the three-dimensional surface of the universe with the added 'halos' even though the galaxies are actually extended outside of the normal three-dimensionality of the universe, as determined by the universal curvature. It would be safe to say that the overall positive curvature of the universe is not strong enough to hold or pull the galactic plane down, thus opening a dimensional gap between the positively curved surface of the universe and the galactic plane, thus leaving the halo as a physical artifact of the gap.

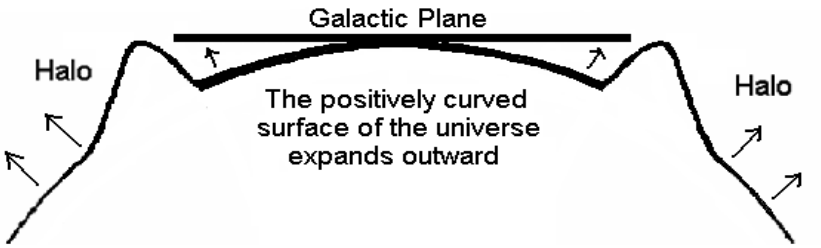
The constant rotational speeds of stars in the arms of spiral galaxies result from a combination of normal gravitational potential due to the dense matter in the core of galaxies and the added energy potential due to the dimensional gap. If the rotational speeds were graphed as a function of the distance of the stars from their galactic center, the normal gravitational forces would provide an exponential curve that decreases as the inverse square of the distance or the increasing galactic radius. However, the contribution from the dimensional gap would graph as a straight line proceeding from the zero point (r=0) to a maximum value of speed at the farthest reach of the galaxy, as shown below.



The actual speeds of stars are the additive product of the central core and the halo's (the dimensional gap's) individual contributions to gravity, as shown by the solid line. The values in this particular graph are for Andromeda. Graphs that look very much like this one have already appeared in text books on astronomy (for example see Hester, et.al., 537), indicating that they have been adopted into mainstream physics, even though this model is the first that has been able to explain them because the source of the halo's gravitational contribution has not yet been identified.

Predictions form the model

As is the scientific norm, this theory must be falsifiable to have any value for science. In other words, the model must have predictive capabilities and it does. Several predictions can be made from this geometrical model and the first deals with the existence of DE itself. Just as the universe expands internally over its three-dimensional volume, it also expands outward in four-dimensional space and in so doing the dimensional gap diminishes over time as it very slowly closes.



However, the dimensional gap manifests itself physically as gravitational potential energy. That energy must go somewhere as the gap closes and the potential decreases or else the conservation of energy would be in danger of failing. Quite simply, that energy bleeds back to the universe as a whole and thus acts to increase the general rate of expansion of the universe. Therefore, the decrease in the dimensional gap and subsequent loss of potential energy from the galaxies is the source of DE. Furthermore, given the rate of expansion, the total number of spiral galaxies in the universe, their average size and mass, this model could be used to predict how much energy is being returned to the universe and thus falsified against the observed value. This calculation requires no more than the Pythagorean Theorem and some simple algebra.

While this model predicts the existence of the increased rate of expansion for the universe that is normally associated with DE, it also predicts a similar more rapid decrease in the expansion rate than average during an earlier period of galactic formation. In this case, the potential energy gained by the galaxies to maintain constant rotational speed during their early growth would come from the universe as a whole in the form of DE, causing a slowing of the expansion rate. This earlier decrease in the expansion rate could account for some discrepancies noted by cosmologists in their speculations, models and researches on the early history of the universe.

Furthermore, since the galaxy appears to be 'pulled down' to fit the overall positive curvature of the universe, the radius of any galaxy would be foreshortened such that the radius of any particular spiral galaxy due to normal gravitational action in the tangent galactic plane would be greater than that observed by astronomers who view the galaxy as if it were only a three-dimensional grouping of stars and star systems that fit the positive curve of the universe. Astronomers would expect the furthest star from the central core of galaxies to be a specific distance from the core that depended upon the total amount of matter in the core, but they would instead observe that the farthest star held in the galaxy by the central core would be slightly closer to the center than expected by normal gravitational considerations. Again, the amount of the apparent foreshortening of projective squashing of galaxies along their radii could easily be calculated by simple geometrical methods and compared to observations.

And finally, the amount of DE in open empty space could be estimated from this model. The maximum dimensional gap for Andromeda, whose graph is used above, has been calculated to be about one-half light year. This calculation was made by applying the Pythagorean Theorem. The amount of kinetic energy in the outermost star of Andromeda could be calculated, since it would be almost solely dependent upon the presence of DM rather than gravitational potential from the matter in the galactic core. From that value and the 'height' of the dimensional gap, the amount of DE filling the gap or in empty space could be estimated. Once the 'width' or 'thickness' in the fourth direction of empty space outside of the galaxies was determined, the amount of DE in any given volume of empty space could be calculated by forming a simple proportionality. At present, the 'thickness' of empty space in the fourth direction is estimated to be approximately the product of the radius of a proton and the fine structure constant, but this value has not been confirmed.

It should be evident that this geometrical model yields simple mathematical values for physical quantities in the form of predictions that render the model falsifiable. No other model of DM is anywhere near ready to make any similar predictions. So we have DM and DE in a nutshell: DM is nothing more than the real extrinsic curvature of the four-dimensional space-time continuum that is not directly associated with local matter, but is a consequence of the non-local material content of the universe that determines the total curvature of the universe. DE is just the field 'thickness' of three-dimensional space in the fourth spatial dimension.

The mathematical model

The major criticism at APS meetings and other professional gatherings where this geometrical model has been presented has been that this model is not mathematical, *i.e.*, there is ***no equation*** to describe the process. And indeed, no equation or other mathematical structure has seemed to emerge directly from the geometrical model. However, an ***equation*** and the subsequent mathematical structure can be derived from outside of the geometrical structure. In particular, if the particulate source of the gravitational forces associated with the DM halo is considered, a mathematical model equivalent to the geometrical model emerges. It is quite evident that the CDM that constitutes the DM halo *could **not** possibly be **particulate***, or rather, separate distinguishable material particles in the DM halo could never amount to the source of the halo itself. This fact follows from a simple logical argument. CDM acts like normal gravitational matter when pumping extra potential energy into the star systems that constitute the outer arms of the spiral. Yet if CDM exhibited normal gravity it would have been attracted to the galactic core during the accretion phase of galactic evolution rather than forming a ring of matter around the galaxy. Therefore, science in confronted with a paradox. The only possibility would be if gravity acted centripetally to hold the CDM to the halo, then the CDM would move to the outer expanses of the galaxy like sliding weights on a rotating pole.

If you place two sliding rings around a long pole with a rubber band stretched between them and then rotate the pole about a center along its length between the two rings, the rings will move outward along the pole in spite of the fact that the rubber band is pulling them radially toward the center of rotation. A simple centripetal force experiment demonstrating this concept is performed in nearly every beginning physics class in the world. Yet this same experiment can explain the DM halo if normal gravity were to have a tangentially directed component corresponding to its normal radially (center) directed force of attraction. This tangential component of gravity would completely account for the DM halo. So a new tangential part of gravity that has never been detected until the effects of the DM halos were observed must exist. A tangential component to gravity of this type would surely indicate the possibility of a fourth spatial dimension, which explains why it has never been detected. This new tangential component of gravitational attraction coupled to the radial component of normal gravity renders normal material particles in the galaxy the source of the DM halo. There is no need to assume any other type of particles or matter exists in the halos.

A gravitational analogy could then be made to the relationship between electricity and magnetism. In other words, we could be faced with a form of gravity and '***gravnetism***' similar to electricity and magnetism. According to electromagnetic theory, curl **A** = **B** and div **A** = 0 where **A** is the electromagnetic vector potential and **B** is the magnetic field strength. By analogy, we would have the new form of curl **W** = $\boldsymbol{\Gamma}$ and div **W** = 0 for gravity, where **W** is a gravitational vector potential and $\boldsymbol{\Gamma}$ is the 'gravnetic' field strength. The 'gravnetic' field results from the total collective force of gravity and thus represents the effect (potential) of the total material content of the universe at any given point in space. It can be argued that both **B** and $\boldsymbol{\Gamma}$ act in four-dimensional space, in that they are four-dimensional quantities, because the 'del' function represents a partial derivative at a point in three-dimensional space and the curl (del cross product) is thus orthogonal to normal three-dimensional space.

The analogies between electromagnetism and gravity can now be carried further, to their conclusion. The total electromagnetic force is defined by the Lorentz equation, just as normal gravity is normally defined by the weight equation:

$$\boldsymbol{F}_{EM} = q\boldsymbol{E} + q\boldsymbol{v} \otimes \boldsymbol{B}$$

and

$$\boldsymbol{F}_{Gr} = m\boldsymbol{g} \, .$$

With the additional tangential component of gravity, we would then have a new pair of equations to define our material world.

$$\boldsymbol{F}_{EM} = q\boldsymbol{E} + q\boldsymbol{v} \otimes \boldsymbol{B}$$

and

$$\boldsymbol{F}_{Gr} = m\boldsymbol{g} + m\boldsymbol{v} \otimes \boldsymbol{\Gamma} \, .$$

These two equations govern ***all*** of the dynamics of the ***whole*** universe, but reduce to a single metrical or kinematical picture of the universe in the Einstein-Kaluza 5-D model. The first terms in the equations, q$\boldsymbol{E}$ and m$\boldsymbol{g}$, are three-dimensional quantities characterized by scalar potentials and the latter terms, q$\boldsymbol{v} \otimes \boldsymbol{B}$ and m$\boldsymbol{v} \otimes \boldsymbol{\Gamma}$, are four-dimensional quantities characterized by vector potentials. The symmetry between gravity and electromagnetism is now complete, but the new equation for gravity must be further investigated and interpreted.

The new equation for gravity mimics the geometrical model for the DM halo. If the equation is rewritten in terms of kinetic energy rather than momentum, the equation takes the form

$$\boldsymbol{F}_{Gr} = m\boldsymbol{g} + 2(KE/v)_{rel}\,\hat{\boldsymbol{r}} \otimes \boldsymbol{\Gamma} \, .$$

Both the kinetic energy KE and the velocity v result from gravity relative to the central core matter of the galaxy. Therefore, the new four-dimensional component of the total speed is linear and increases as the relative velocity decreases or the distance (radius) from the galactic center increases. The second term would graph as a straight line coinciding with that of the potential gap while the first term representing normal gravity would graph as above with speed decreasing as the inverse square relative to the radius of the galaxy. So this formula perfectly mimics the geometrical theory of the DM halo as stated above. However, the source of DE can now be identified more precisely as the gravitational potential vector **W**. When the vector potential interacts with matter and space it appears as a potential energy, thus DE. Also, the physical reality of vector potentials in general is only implied by their mathematical existence such that they cannot be measured in three-dimensional space. The electromagnetic vector potential, literally a magnetic potential, has never been directly detected or measured, but its existence has been verified experimentally by the Aharonov-Bohm and other experiments. While it is a vector potential relative to three-dimensional space, it cannot be measured in three-dimensional space because the length of the vector points in the fourth direction of space. So the vector potentials, both electromagnetic and gravitational, correspond to lengths in four-dimensional space and the gravitational vector, in particular, thus determines the dimensional gap that develops as a galaxy accretes matter to form its core.

Furthermore, since $\boldsymbol{\Gamma}$ is the external 'gravnetic' field corresponding to the net gravity of the universe and m$\boldsymbol{v}$ represents the 'gravnetic' field due to the matter in a galaxy (or an individual object), they would always add together to create the halo since gravity is only attractive. In this model, m$\boldsymbol{v}$ could also be interpreted as representing the

inertia of moving bodies relative to $\boldsymbol{\Gamma}$, the rest of the universe. Therefore, the quantity $m\boldsymbol{v} \otimes \boldsymbol{\Gamma}$ would amount to a mathematical form of Mach's Principle. This principle has been known for more than one hundred years and is an integral part of modern physics and cosmology, but has never before been expressed in a mathematical form. Under these circumstances, it can be restated: The relative velocity $\boldsymbol{v}$ could never be zero with respect to each and every bit of matter in the universe since that would constitute an absolute reference frame, *i.e.*, a frame at rest with respect to all matter in the universe. Therefore, there is no absolute space.

Mach's Principle deals directly with the concept of inertia. In that case, the local inertia of a material body would depend upon its state of motion as in Newton's first law of motion. And indeed, inertia does depend upon the motion of a body and increases with the relative speed of the body according to the formulations of special relativity. Under these circumstances, this model of DM, DE and the material universe allows science to define inertia relative to the whole universe, represented by $\boldsymbol{\Gamma}$, as a four-dimensional property of space. $\boldsymbol{\Gamma}$ is constant since gravity is always additive, even though normal gravity follows the inverse square law, and can never be blocked by its opposite, which does not exist. This is the same as saying that the total amount of matter in the universe is constant. This fact guarantees that the rest mass and inertia of any and every material object should remain constant, as it in fact does, because the inertia and rest mass are dependent upon $\boldsymbol{\Gamma}$, which is constant.

The units for $\boldsymbol{\Gamma}$ would be 1/sec, which would seem to make $\boldsymbol{\Gamma}$ a frequency, while $\boldsymbol{\Gamma}$ is constant throughout the universe. Waves have frequencies; material bodies do not. So the net gravity of the universe would appear to any given material particle or body as a constant wave represented by the frequency $\boldsymbol{\Gamma}$. Gravity waves are predicted by general relativity and as such would exhibit a frequency, so the quantity $m\boldsymbol{v} \otimes \boldsymbol{\Gamma}$ would indicate that the inertia of moving bodies in a galaxy ($m\boldsymbol{v}$) would interact with the gravity field of the rest of the universe ($\boldsymbol{\Gamma}$) via gravitational waves. So DM halos could also be interpreted as either a superposition of real waves or standing gravitational waves in four-dimensional space. The amplitude of the standing gravitational wave would equal the dimensional gap at any point in a galaxy or beyond its outer limit. Given this interpretation, the concept of a 'macroscopic' distance in the new fourth direction of space must also be reinterpreted.

In reality, the concept of distance is 'irrelevant' with respect to the fourth dimension. Distance is a strictly three-dimensional quantity derived from the relative position of three-dimensional material bodies. Yet all three-dimensional material bodies must only occupy a single and unique three-dimensional 'slice' or 'sheet' in the overall four-dimensional embedding space that is characterized by the magnetic and 'gravnetic' fields. In other words, there are no material objects in the fourth dimension other than those that constitute the three-dimensional 'sheet' of our normal material existence. With no separate material bodies in the fourth dimension, outside of the three-dimensional 'slice' of four-dimensional space, there is no way to measure a relative distance in the fourth dimension of space. So any concept of normal distance in the fourth dimension is without foundation and the concept of 'compactification' of higher dimensions that is touted by some physical theorists is completely unnecessary and irrelevant. The concept of 'compactification' was only invented as a mathematical ploy and gimmick to explain why higher dimensions were not detected or sensed. In reality, they are not detected or sensed because distances in the higher dimension or dimensions are irrelevant and have no meaning relative to our three-dimensional space of experience. The fourth dimension has only been considered 'macroscopic' in this model and theory because the fourth dimension of space has been expanding since the big bang, just like the normal three dimensions of space, in spite of the irrelevance of distance in that dimension.

And finally, this new mathematical model offers a simple path or guide to developing a new model of quantum gravity. By using deBroglie's concept of a matter wave given by the equation $\lambda_{matter} = h/mv$, the equation for

gravitational force $\boldsymbol{F}_{Gr}$ can be restated in quantum terms. From deBroglie's equation we get mv = h/λ, but also v = fλ so that mv = hf/v and therefore

$$\boldsymbol{F}_{Gr} = m\boldsymbol{g} + 2(hf/v)\,\hat{\boldsymbol{r}} \otimes \boldsymbol{\Gamma} \ .$$

Gravity is thus quantized without reference to a 'graviton' as has been the practice of previous theoreticians seeking a quantum theory of gravity. From a philosophical point of view, transverse gravitational waves would not be like other waves because transverse waves in three-dimensional space have a negative and a positive portion. But gravity is never negative and associating 'gravitons' with three-dimensional transverse gravity waves, as is the practice in quantum theory, will just not work. According to this model, the portion of gravity that is quantized is the amplitude of the gravity wave in the fourth direction of space, which would correspond to the amplitude of gravitational waves and the concept of a potential vector that extends into the higher fourth dimension.

Conclusion

The strengths of this theory of DM and DE should be quite evident: The theory is simple, straight forward, explains the phenomena and is completely falsifiable at several levels. It is also extremely flexible with room for expansion. Moreover, it offers a great deal of new physics that goes beyond both DM and DE. The concept of a real physical fourth dimension of space is revolutionary even though it is more than a century old. If this theory or the basic hypothesis is criticized for being either non-mathematical or not mathematical enough then the critics should look at the wealth of highly mathematical material on four-dimensional spaces and five-dimensional space-times that is already available. This model merely draws together the work of others that has been ignored for decades.

In fact, one particular scientist published more than thirty papers in peer reviewed journals on the unification of general relativity and the quantum theory using a five-dimensional Einstein-Kaluza framework between the 1920s and the 1960s. He worked in collaboration with others, but most of his articles came from his mind alone. He derived the Dirac equation and accounted for the Yukawa Potential in his basic five-dimensional model, but his work has been soundly ignored by the scientific community at large. His name would not even be recognized by most scientists: Nor would the names of many other researchers working on similar theories and models. This particular scientist even published a book summarizing his complete theory of unification in 1966, after he officially retired from a long career of teaching and theoretical research. While numerous physicists try to unify physics under a single paradigm today, they would never recognize the name of Henry T. Flint and the others who have already gone a long way toward that very unification. However, the discoveries of DM and DE and their explanation by this theory, as stated above, should finally wake a sleeping scientific community to our higher dimensional reality. This is not a conclusion, but the beginning of a new physics.

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